A Simplified Mechanistic Model for an Oil/Water Horizontal Pipe Separator

Eduardo Pereyra, Ram Mohan, and **Ovadia Shoham,** University of Tulsa

Summary
A new methodology for oil/water horizontal pipe separator (HPS) design and performance prediction is developed. The separator diameter is determined on the basis of oil/water flow-pattern prediction. A batch separator model is adopted and modified for pipe flow to predict the separator length for achieving a desired separation quality. An experimental program is carried out to validate the proposed model.

Introduction
For oil/water mixtures in horizontal pipes, the occurring flow patterns can be classified as dispersed and segregated. For the segregated flow pattern, the phases can be separated by gravity because of the lower velocities of the phases. This phenomenon allows the use of a simple horizontal pipe as an oil/water separator, namely the horizontal pipe separator (HPS). The HPS can be used as an attractive alternative to conventional separation systems.

The incoming mixture consists of one defined continuous phase and one defined disperse phase. The total volume of drops entering the separator is equal to the incoming dispersed-phase flow. A schematic of the oil/water separation process for water-continuous flow in a HPS is presented in **Fig. 1.** For this case, the oil enters into the separator as dispersed phase. The turbulence occurring at the entrance of the separator keeps the oil droplets dispersed in the pipe cross-sectional area, delaying the formation of the oil and/or the dense-packed layers at the upper part of the pipe. The dense-packed layer is characterized by oil droplets separated only by a thin water/liquid film. Below this layer, droplets are sparse, moving upward because of gravity, forming the sedimentation layer. Finally, the free water layer occurs at the bottom of the pipe. Both the oil and the water layers increase in height as the flow moves downstream. The HPS is proposed to operate only in laminar flow; turbulent flow hinders separation, because turbulent fluctuations promote mixing of the phases, resulting in a lower separation efficiency.

Wang et al. (2006) used the HPS in an integrated compact multiphase separation system, designed for free water knockout. Field tests showed satisfactory performance of the HPS for a wide range of operational conditions. The authors also state the need for a design methodology and performance model for the HPS to improve the entire integrated system.

Perez (2005) evaluated the HPS as an oil/water separator. He investigated experimentally and theoretically the developing region of oil/water stratified flow in a horizontal pipe. Local velocity profile, water fraction, and droplet-size distribution measurements were reported. A mechanistic model was developed for the HPS on the basis of two submodels, namely the hydrodynamic and the coalescence models. The hydrodynamic model considers the momentum transfer between three layers, including the water-continuous, dense-packed, and oil-continuous layers. The coalescence submodel uses simplified population-balance equations to account for the droplet evolution within the dense-packed zone. This model requires as an input several coalescence parameters of the oil/water mixture, which cannot be obtained from a simple test. A simplified mechanistic model wherein the coalescence parameters can be determined by a simple test is presented in this study. The proposed model can be used for proper design and performance evaluation of the HPS.

Modeling
Determination of two parameters are required for the HPS design, namely pipe diameter (ID) and length (L). The first parameter is defined as the required diameter to promote stratification of the phases, while the second parameter is the necessary length to achieve a desired separation quality.

Because of the low liquid velocities, the momentum of the continuous phase is low and gravity dominates the separation process. Thus, the behavior of the HPS can be approximated by an extension of the batch-separation process. Batch-separator models predict the evolution of the separation profiles as a function of time. These models are extended to the HPS by considering that both phases move at same velocity, namely the mixture velocity u_M. This assumption neglects any velocity profile and momentum exchange between layers that affects the droplet coalescence.

Separation Profiles. Among the available theories for batch-separator behavior prediction, the asymmetric dimple model pre-

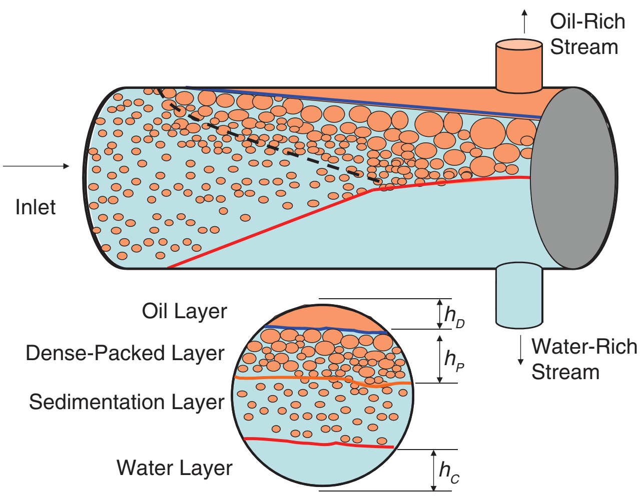

Fig. 1—Schematic of HPS for water-continuous flow.

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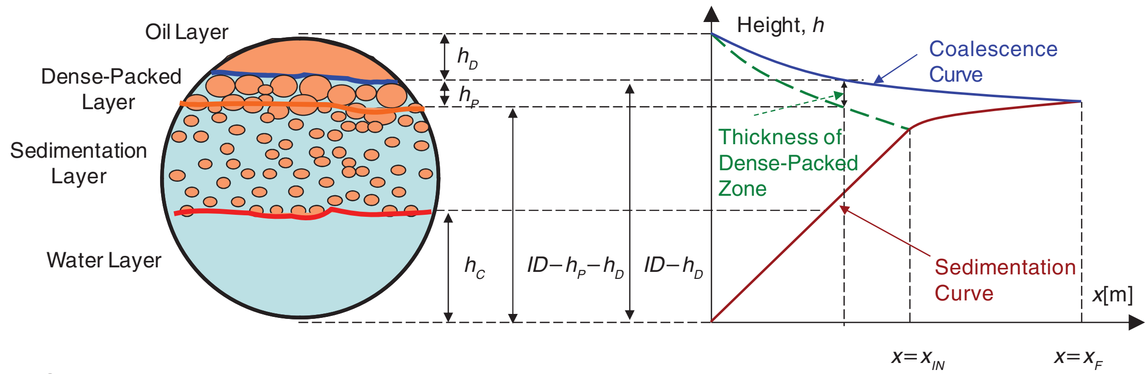

Fig. 2—HPS settling curve for water-continuous dispersion.

sented by Henschke et al. (2002) is the most suitable for extension to the HPS. This model requires calibration of a unique parameter, which is related to the dimensionless asymmetry of the film drainage r_V*. As reported by the author, this parameter r_V* is independent of the initial droplet-size distribution, water cut, or batch height for the same oil/water mixture. This capability facilitates the scaleup of a simple bottle test to field-scale conditions. The Henschke et al. (2002) batch-separator model is extended to pipe flow in the HPS by changing the timescale in the batch separator to space using the average oil/water mixture velocity u_M. The main assumptions of this model are as follows:

1. Mono dispersion (all droplets have the same size at a given time and position).
2. Clean interface or negligible surface agent effect.
3. Asymmetrical film drainage.

As can be seen in **Fig. 2,** the separation profile can be predicted by determining the evolution of the three layers of thickness, namely, the single-phase h_D, dense-packed h_P, and sedimentation h_C layers. For the HPS, the cross-sectional area of the pipe varies in the vertical direction; thus, the model is defined as a function of volumes, instead of levels. Geometrical relationships are required to relate levels to cylinder volumes, which are described next.

Sedimentation Analysis. Gravity sedimentation and flotation are the first separation mechanisms acting on an oil/water mixture. Several dimensionless parameters were presented by Pilhofer and Mewes (1979) in order to include the droplet internal flow circulation. These parameters include the Archimedes number Ar, friction coefficient C_w, Hadamard-Rybczynski factor K_{HR}, Reynolds number of a single droplet in an infinite fluid Re_∞, the dispersed-phase viscosity and the parameters ξ and λ. Following is the proposed sedimentation analysis:

$$v_S = \frac{3\lambda\phi\mu_C}{C_w\xi(1-\phi)\rho_C d_P}\left[\left(1+Ar\frac{C_w\xi(1-\phi)^3}{54\lambda^2\phi^2}\right)^{0.5}-1\right], \quad (1)$$

and the considered parameters are defined as

$$\mathrm{Ar} = \frac{\rho_C \Delta\rho g d_P^3}{\mu_C^2} \quad (2)$$

$$\xi = 5K_{HR}^{-3/2}\left(\frac{\phi}{1-\phi}\right)^{0.45} \quad (3)$$

$$K_{HR} = \frac{3(\mu_C+\mu_D)}{2\mu_C+3\mu_D} \quad (4)$$

$$\lambda = \frac{1-\phi}{2\phi K_{HR}}\exp\left(\frac{2.5\phi}{1-0.61\phi}\right) \quad (5)$$

$$C_w = \frac{\mathrm{Ar}}{6\mathrm{Re}_\infty^2} - \frac{3}{K_{HR}\,\mathrm{Re}_\infty} \quad (6)$$

and

$$\mathrm{Re}_\infty = \frac{\rho_C v_S^\infty d_P}{\mu_C} = 9.72\left[(1+0.01Ar)^{4/7}-1\right] \cdot \quad (7)$$

The evolution of the sedimentation curve can be predicted by Eq. 1 for a given mixture of fluid properties as follows:

$$h_C = \frac{v_S\, x}{u_M} \cdot \quad (8)$$

The average droplet diameter for the settling velocity is given by the Sauter mean diameter at the inlet of the separator d_{32}^0.

Coalescence Analysis. The oil droplets move to the top of the pipe where coalescence occurs, forming the oil layer with thickness h_D. The rate of change of h_D with respect to time is determined by the coalescence of the droplets with a flat interface. Assuming that all the droplets at the interface have the same diameter d_{32}^I and using the geometry presented in **Fig. 3,** Pereyra (2011) demonstrated that the rate of change of the homophase circular-cross-sectional area is given by

$$u_M\frac{dh_D}{dx} = \frac{2\,\phi_I\, d_{32}^I}{3\tau_I} \cdot \quad (9)$$

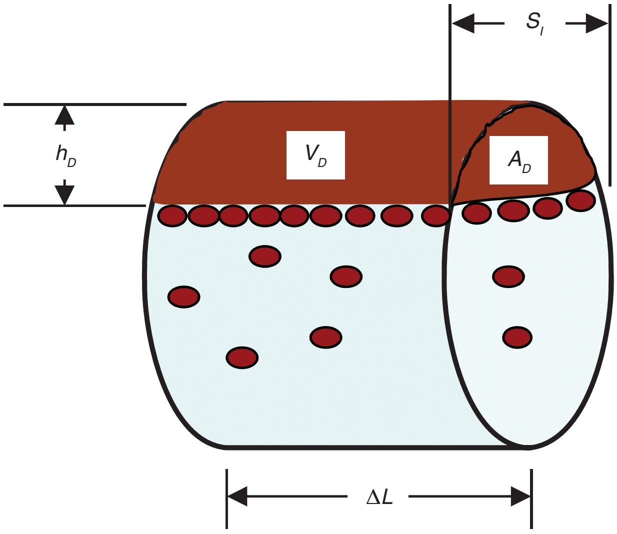

Fig. 3—Schematic of disengaging interface.

The dispersed-phase fraction at the interface ϕ_I is approximately equal to the unit (1). Eq. 9 defines the evolution of the single-phase layer h_D as a function of time, which can be solved numerically by an explicit scheme. The solution requires determination of the Sauter mean diameter at the interface, which is presented next.

Binary coalescence or coalescence between two particles is considered at the dense-packed region. Assuming that, during each timestep and at the same vertical position, all droplets have the same size and equal to the Sauter mean diameter d_{32}, Hartland and Jeelani (1988) proposed the following expression for predicting the evolution of a droplet size as a function of the droplet-droplet coalescence time τ_C:

$$u_M \frac{d(d_{32})}{dx} = \frac{d_{32}}{6\tau_C}. \quad (10)$$

Eq. 10 enables determination of the evolution of droplet sizes in the dense-packed region. Initially, the droplets are uniformly distributed with size d_{32}^0 along the cross-sectional area. Also, the droplet size at the interface (d_{32}^I) is given by the location of the single-phase layer. Discretization in time and space is required for the solution of Eq. 10.

The evolution of the dense-packed zone height h_P (see Fig. 1) is determined by a mass balance at the cross-sectional area. For an oil-in-water dispersion with an initial dispersed-phase fraction of ϕ_0 there exists an axial distance x_{IN} where the droplet sedimentation is completed and the height of the dense-packed zone (h_D-h_P) starts to decrease (see Fig. 2). Upstream of this location ($0<x<x_{IN}$), the area of the dense-packed zone is given by

$$A_P = \frac{A_C\,\phi_0 - (1-\phi_0)A_D}{\phi_{P0} - \phi_0}. \quad (11)$$

After the sedimentation process is completed ($x>x_{IN}$), the area of the dense-packed zone is expressed as

$$A_P = \frac{A_S\phi_0 - A_D}{\bar{\phi}_P}. \quad (12)$$

The average holdup in the dense-packed zone ($\bar{\phi}_P$), as a function of the axial position, can be estimated as follows:

$$\bar{\phi}_P = \phi_I - \exp\left(-C_1\frac{x}{u_M} - C_2\right), \quad (13)$$

where the coefficients C_1 and C_2 can be determined on the basis of continuity. At the inflection point $x=x_{IN}$, $\bar{\phi}_P=\phi_{P0}$, and the slopes of the sedimentation curve (before the inflection point) and the dense-packed zone (after the inflection) are equal, yielding

$$C_1 = \frac{\bar{\phi}_{P0}{}^2\psi}{(A_S\,\phi_0 - A_D)(\phi_I - \phi_{P0})}$$
$$C_2 = -C_1\frac{x_{IN}}{u_M} - \ln(\phi_I - \phi_{P0}), \quad (14)$$

where

$$\psi = \left[\begin{matrix}\dfrac{\partial(A_P)}{\partial h_P}\left(v_S + u_M\dfrac{d(h_D)}{dx}\right) - \dfrac{u_M}{\bar{\phi}_{P0}}\dfrac{\partial(Ad)}{\partial hd}\dfrac{d(hd)}{dx} \\ -u_M\dfrac{\partial(Ap)}{\partial hd}\dfrac{d(hd)}{dx}\end{matrix}\right]_{x=x_{IN}}. \quad (15)$$

Coalescence Time. The droplet-size distribution in the dense-packed zone is required for modeling the coalescence process. The higher the dense-packed zone, the stronger the deformation of the droplets, which affects the coalescence process. On the basis of a numerical solution of droplet deformation in a dense-packed zone, Henschke et al. (2002) developed an empirical formulation for droplet deformation, where the radius of the droplet/droplet contact area $r_{F,C}$ is given by

$$r_{F,C} = 0.3025\,d_{32}\sqrt{1-\frac{4.7}{\mathrm{La}+4.7}}. \quad (16)$$

For droplet to interface coalescence, the contact area radius $r_{F,I}$ is given by

$$r_{F,I} = \sqrt{3}\,r_{F,C}. \quad (17)$$

The radius of the channel contour can be predicted from

$$ra = 0.5\,d_{32}\left(1-\sqrt{1-\frac{4.7}{\mathrm{La}+4.7}}\right). \quad (18)$$

A modified Laplace number was suggested by Henschke et al. (2002) as follows:

$$\mathrm{La} = \left(\frac{|\rho_D - \rho_C|\,g}{\sigma}\right)^{0.6} h_P^{\,0.2}\,d_{32}. \quad (19)$$

Based on an asymmetric film-drainage analysis, the authors proposed the following equation for the coalescing time of two droplets with diameter d_{32}:

$$\tau_C = \frac{(6\pi)^{7/6}\mu_C\,r_a^{\,7/3}}{4\sigma^{5/6}\mathrm{H}^{1/6}r_{F,C}\,r_v^*}. \quad (20)$$

The coalescing time of one droplet, with diameter d_{32}, with a flat interface is given by a modification of Eq. 20 as follows:

$$\tau_I = \frac{(6\pi)^{7/6}\mu_C\,r_a^{\,7/3}}{4\sigma^{5/6}\mathrm{H}^{1/6}r_{F,I}\,r_V^*}. \quad (21)$$

Eq. 21 contains two unknown parameters, namely the Hamaker coefficient H and the asymmetry parameter r_V*. The asymmetry parameter is adjusted with experimental settling curves, and thus it is characteristic for the system used. Henschke et al. (2002) proposed a Hamaker coefficient equal to 10^{-20} Nm for all systems.

As mentioned, the solution of these models required discretization in space and time. Using Eq. 10, the spatial discretization allows the prediction of Sauter mean diameter along the dense-packed zone, while time discretization is used to predict the evolution of the layers' thickness as a function of time. The Appendix contains the remaining closure relationships for solving the proposed model.

HPS Length Determination. The total length of the separator can be determined from the intersection point between the coalescence curve h_D and the sedimentation curve h_C. Usually, a complete separation requires an impractically long length. Additionally, the main objective of the HPS is water knockout, which can be achieved with shorter length. Thus, a different design criterion is proposed, considering the required separator length for achieving a desired water fraction in the oil-rich stream (Wc_{WRS}), which can be predicted by

$$Wc_{WRS} = \frac{A_C + (SR\,A_S - A_C)(1-\phi_{(x)})}{SR\,A_S}. \quad (22)$$

The split ratio (SR) or the fraction of the inlet mixture flow rate q_M flowing in the water-rich stream q_{WRS} is given by

$$SR = \frac{q_{WRS}}{q_M}. \quad (23)$$

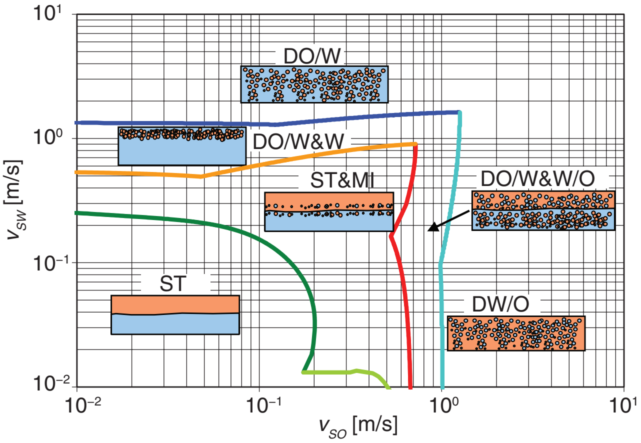

Fig. 4—Trallero (1995) oil/water flow-pattern classification.

For a given *SR*, Eq. 22 can be used to predict the performance of an existing HPS separator.

HPS Diameter Determination. Different flow-pattern configurations occur in horizontal oil/water flow. Trallero (1995) proposed a flow-pattern classification for fully developed oil/water flow in horizontal pipes **(Fig. 4)** as follows:

1. Stratified flow (*ST*): This flow pattern is characterized by two liquid layers with the heavier (usually water) at the bottom and the lighter (usually oil) at the top.
2. Stratified flow with mixing at the interface (*ST* and *MI*): For this pattern, the system tends to be stratified; however, interface instability generates a mixing zone. The mixing zone at the interface can be significant, but still pure fluids exist at the top and the bottom of the pipe.
3. Dispersion of oil in water with a water layer (*DO/W* and *W*): The water is distributed across the entire pipe. A layer of clean water flows at the bottom and dispersed droplets of oil in water flow at the top.
4. Dispersion of oil in water (*DO/W*): In this case, the entire pipe cross-sectional area is occupied by water containing dispersed oil droplets.
5. Dispersion of water in oil (*DW/O*): The oil is the continuous phase and the water is present as droplets across the entire pipe cross-sectional area.
6. Dual-dispersion (*DO/W* and *W/O*): In this flow pattern, two different layers occur. Both phases are present across the entire pipe, but at the top the continuous phase is oil, containing droplets of water. In the lower region of the pipe, the continuous phase is water and the oil exists as dispersed droplets.

Torres-Monzon (2006) presented the most recent flow-pattern prediction model for oil/water flow in horizontal pipes. The proposed model simplifies significantly the flow-pattern map for liquid/liquid flow, showing a fair agreement with a large number of experimental data sets. On the basis of the Torres-Monzon (2006) model, the HPS diameter is determined as the smallest nominal diameter, for which stratified flow (*ST*) is predicted.

Experimental Program

An experimental program has been carried out to validate the prediction of the separation profiles in the HPS. The multiphase flow loop located at the north campus of the University of Tulsa was used in this study. This indoor facility **(Fig. 5)** is a fully instrumented state-of-the-art flow loop. A transparent PVC pipe was used to enable measurement of the evolution of the different layers in the separator. The experimental setup consists of four major sections: a storage and metering section, a HPS test section, a multiphase separation section, and a data-acquisition system. The flow rates and densities of both water and oil are measured using the Micromotion mass flowmeters. The oil and water are combined in a mixing tee to obtain an oil/water mixture. A static mixer (KOMAX CPS), in series with the mixing tee, is installed to promote mixing of the two liquids. The HPS body is a 0.1-m ID, 6-m.-long transparent PVC pipe, built from pipe spools. The oil and water outlet pipe sections are 0.0254 m in diameter. The outlets are connected directly (abrupt contraction) to the separator body **(Fig. 6).**

Tap water and mineral oil (Tulco Tech 80) have been used as working fluids. The oil density and viscosity are 857 kg/m^3 (at 15.6°C) and 13.6 m Pa s (at 37.8°C), and the interfacial tension is 0.029 N/m.

The experimental test matrix used has been in the steady-state, stratified flow-pattern region. The Torres-Monzon (2006) model was considered to generate the flow-pattern map for the HPS con-

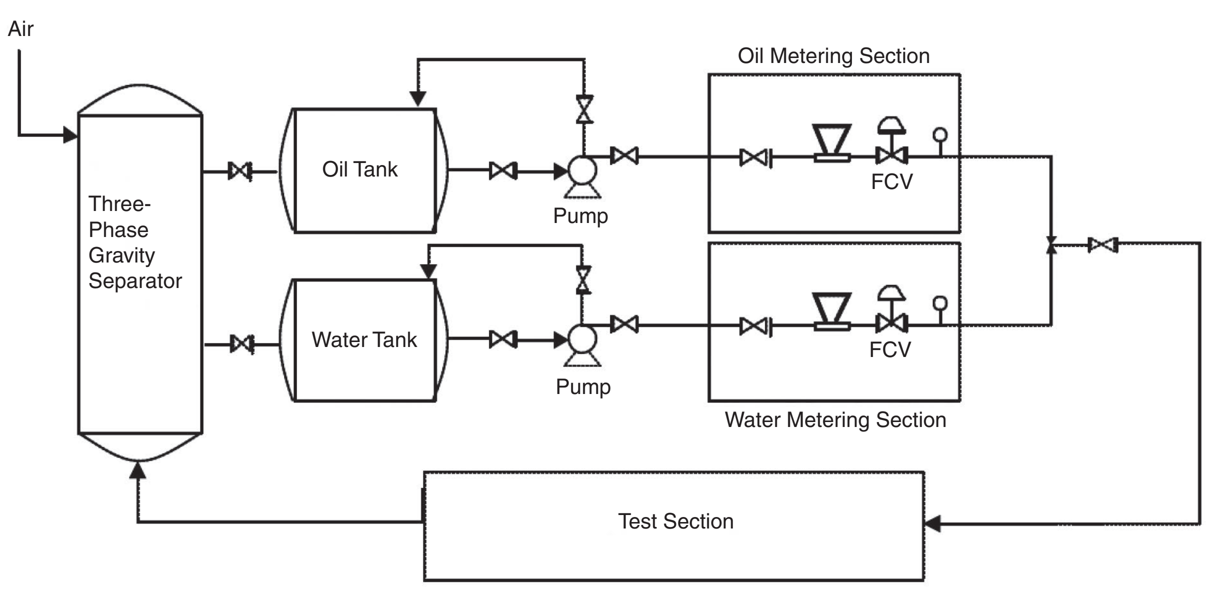

Fig. 5—Schematic of HPS experimental facility.

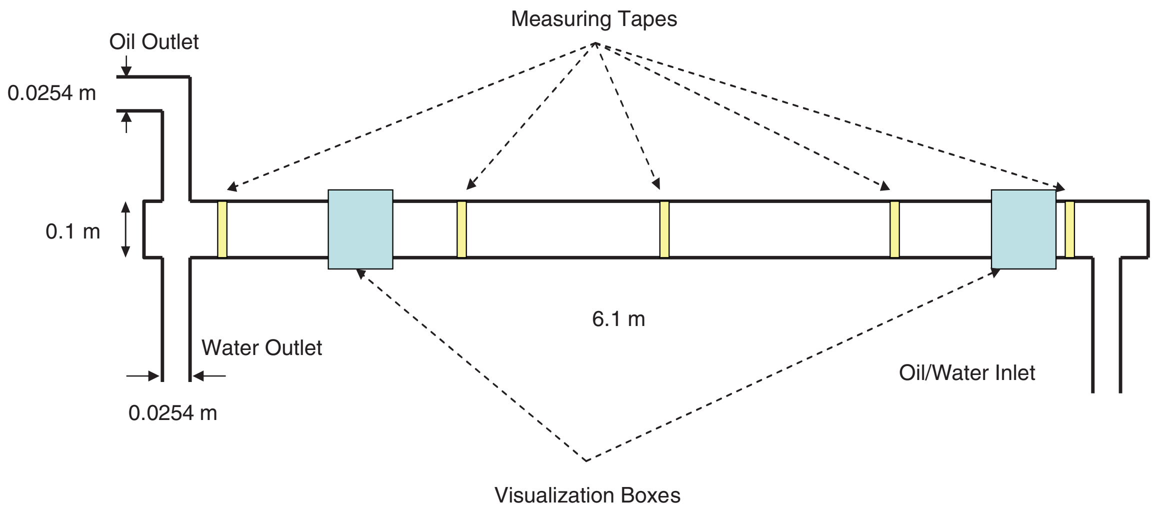

Fig. 6—HPS measuring tapes and visualization boxes.

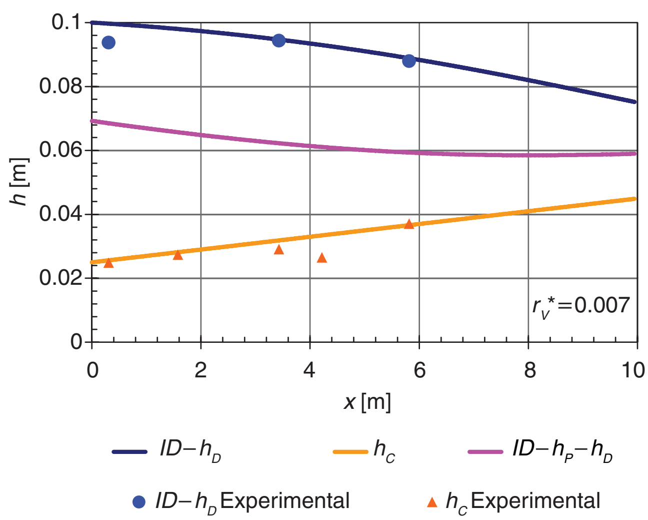

Fig. 7—Comparison between model prediction and experimental data for W_c=60% and u_M=0.09 m/s.

ditions. Two different water cuts have been evaluated (40 and 60%) for different mixture velocities (0.06, 0.09, and 0.13 m/s).

The thickness of the water and oil layers were measured in five different locations along the clear pipe using measuring tapes (Fig. 6). The reading of the measuring tapes was corrected to include the effect of the pipe curvature. A known volume of liquid has been added to the separator, recording the resulting internal liquid level and wetted perimeter with tapes. A calibration curve, relating the wetted perimeter (measure-tape readings) with the real liquid level, is used to measure the layers' thickness. Using only visual observation, it is difficult to identify the interface between the dense-packed zone and the sedimenting region. Thus, the dense-packed zone thickness has not been measured in this study.

Model Validation and Design Example

The acquired experimental data are used to validate the proposed model, which requires calibration of the asymmetric dimple parameter r_V*. As presented in **Fig. 7,** the asymmetric dimple parameter for 60% water cut and a mixture velocity of 0.09 m/s is r_V*=0.007. This asymmetric parameter is used for the prediction of the separation profiles for two different mixture velocities, namely u_M=0.06 and u_M=0.13 m/s with 60% water cut. The model and experimental data exhibit fair agreement, as depicted in **Figs. 8 and 9,** respec-

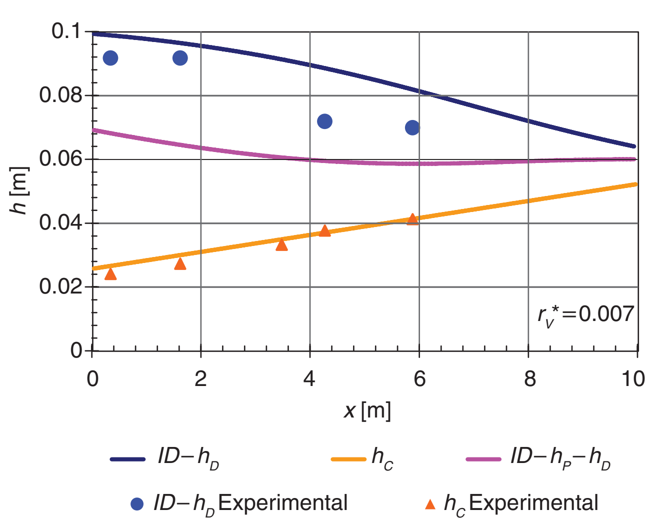

Fig. 8—Comparison between model prediction and experimental data for W_c=60% and u_M=0.06 m/s.

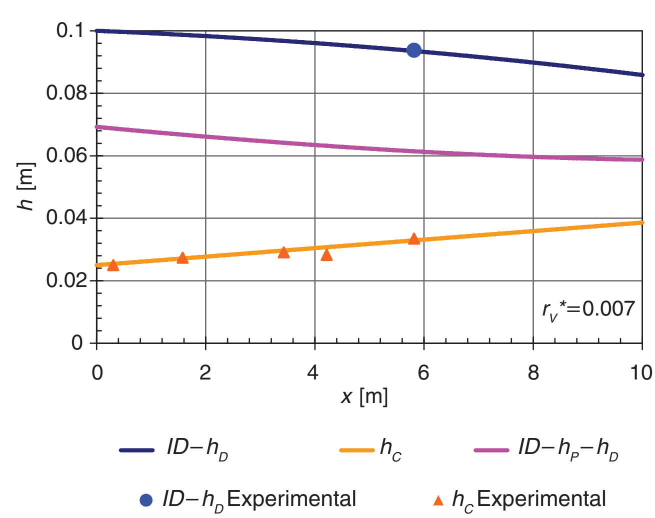

Fig. 9—Comparison between model prediction and experimental data for W_c=60% and u_M=0.13 m/s.

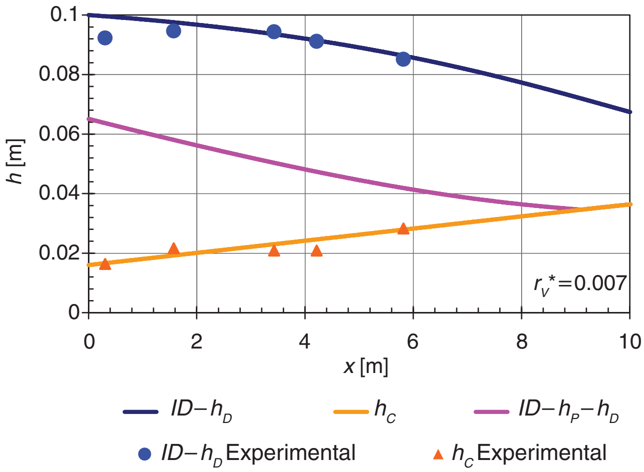

Fig. 10—Comparison between model prediction and experimental data for W_c=40% and u_M=0.09 m/s.

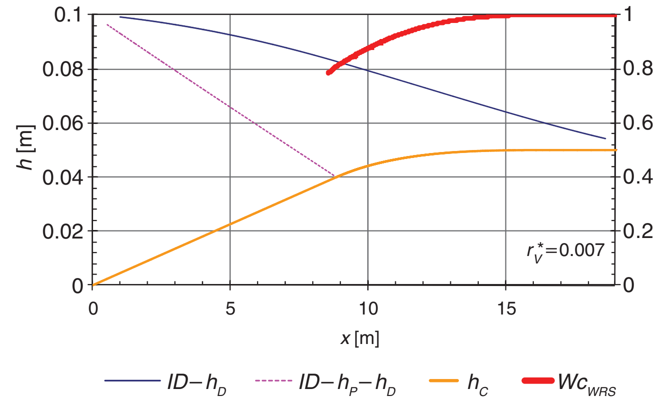

Fig. 11—Water fraction in the water-rich stream vs. axial position.

tively. Finally, keeping the same mixture velocity of u_M=0.09 m/s and reducing the water cut to 40%, the results are plotted as shown in **Fig. 10.** The asymmetric parameter r_V* is maintained constant for same fluid system at different flow conditions. As demonstrated, the proposed model can be used to scale-up laboratory tests for the same oil/water mixture. Further studies are required to develop a universal closure relationship for the asymmetric parameter considering different oil/water mixtures.

HPS Design Example. For a mixture of mineral oil (Tulco Tech 80) and tap water flowing in a horizontal pipe at u_M=0.091m/s and W_c=0.5, a 0.1-m diameter is predicted for proper stratification. The separator length is determined by the required distance to achieve Wc_{WRS}=0.96 with a SR=0.5, considering an initial droplet size of d_{32}^0=250 µm. **Fig. 11** presents the water cut evolution in the water-rich stream (Wc_{WRS}) as a function of the axial position. For this case, the experimental asymmetric dimple parameter of the mixture is r_V*=0.007. For these conditions, the predicted length of the separator is 13 m, which is considerably longer than the experimental facility. As can be seen, the experimental data never show full separation owing to limited length (L=6 m) as compared with the results presented in Fig. 11.

Conclusions

On the basis of this study, the following conclusions are reached:

1. A new HPS model and design methodology are proposed based on the hydrodynamic flow behavior of the oil/water mixture in the separator.
2. The minimum HPS diameter can be determined on the basis of flow-pattern prediction to ensure stratified flow in the separator.
3. The evolution of the three layers of thickness, namely the single-phase, dense packed, and sedimentation layers, are predicted by extension of a batch-separator model, assuming constant mixture velocity in the separator, and incorporating the pipe geometry.
4. Prediction of the HPS length requires the calibration of a single asymetric dimple parameter. This can be carried out by a simple bottle test of the oil/water mixture. This parameter is independent of droplet size, water cut, and height, allowing the scale-up of simple bottle tests to field conditions.
5. The proposed model can be used for the prediction of water fraction at the HPS outlet.

Nomenclature

A = area, m²
Ar = Archimedes number
d = droplet diameter, m
d_{32} = Sauter mean diameter, m
C_1 = constant
C_2 = constant
C_w = friction coefficient
g = gravity, m/s²
h = layer thickness, m
H = Hamaker coefficient
ID = pipe or separator diameter, m
K_{HR} = Hadamard-Rybczynski factor
L = pipe or separator length, m
La = modified Laplace number
q = flow rate, m³/s
$r_{F,C}$ = radius of the droplet/droplet contact area, m
$r_{F,i}$ = Contact area radius, m
r_V* = Dimensionless asymmetry of the film drainage parameter
ra = radius of the channel contour, m
Re_∞ = Reynolds number of a single droplet in an infinite fluid
S = perimeter, m
SR = split ratio or the fraction of inlet mixture flow rate flowing into water reach stream
t = time, s
V = volume, m³
v_S = slip velocity, m/s
u = axial velocity, m/s
W_c = water cut
x = axial position, m
λ = settling velocity parameter
μ = viscosity, Pa s
ξ = settling velocity parameter
ρ = density, kg/m³
τ_C = droplet/droplet coalescence time
τ_I = droplet interface coalescence time
ϕ = dispersed phase fraction

Subscripts

C = continuous phase or continuous-phase layer
D = dispersed phase or dispersed-phase layer
I = interface
IN = inflection point or point where sedimentation is completed
M = mixture
O = oil
P = dense-packed layer
S = separator

W = water
WRS = water-rich stream

Superscripts

∞ = infinite medium
0 = initial or separator inlet
I = interface

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Appendix A—Closure Relationships

The cross-sectional area occupied by the homophase (A_D) is determined from Eq. 9, while the dense-packed zone area (A_P) can be predicted by Eqs. 11 and 12. Closure relationships between layer thickness and area are required to solve the equations. The sedimentation area (A_C) is given by

$$A_C = \frac{ID^2}{4}\left[\pi - \cos^{-1}(\omega_C) + (\omega_C)\sqrt{1\text{-}(\omega_C)^2}\right] \quad \text{(A-1)}$$

where $\omega_c = 2h_c/\mathrm{ID} - 1$. The homophase area ($A_D$) and its derivative are

$$A_D = \frac{ID^2}{4}\left[\pi - \cos^{-1}(\omega_D) + (\omega_D)\sqrt{1\text{-}(\omega_D)^2}\right]$$
$$\frac{d(A_D)}{d\,h_D} = 2\sqrt{h_D\ (I\mathrm{D} - h_D)}, \quad \text{(A-2)}$$

where $\omega_D = 2h_D/\mathrm{ID} - 1$. The area of the dense-packed zone and its partial derivatives with heights can be predicted by

$$A_P = \frac{ID^2}{4}\left[\pi - \cos^{-1}(\omega_P) + (\omega_P)\sqrt{1\text{-}(\omega_P)^2}\right] - A_D$$
$$\frac{\partial(A_P)}{\partial\,h_P} = 2\sqrt{(h_D + h_P)(I\mathrm{D} - h_P - h_D)} \quad \text{(A-3)}$$
$$\frac{\partial(A_P)}{\partial\,h_D} = 2\sqrt{(h_D + h_P)(I\mathrm{D} - h_P - h_D)}$$
$$-2\sqrt{h_D\ (I\mathrm{D} - h_D)},$$

where $\omega_D = 2h_D/\mathrm{ID} - 1$.

Eduardo Pereyra is a research associate with the Fluid Flow Project at the University of Tulsa. His research interests include multiphase flow systems and transport, flow assurance, and separation technologies. Pereyra has appeared in several refereed journals and has written conference papers in these areas. Pereyra holds two BE degrees, one in mechanical engineering and one in systems engineering, from the University of Los Andes, Venezuela, and MS and PhD degrees in petroleum engineering from the University of Tulsa.

Ram S. Mohan is a Professor of Mechanical Engineering at the University of Tulsa. Mohan teaches and conducts research in the areas of multiphase flow, oil/water dispersion, instrumentation and measurements, control systems, compact separators, computer-aided design, and manufacturing processes. He currently serves as the co-director of Tulsa University Separation Technology Projects (TUSTP), supported by several oil companies. Mohan also directs several projects supported by the Chevron TU Center of Research Excellence (TU-CoRE). Mohan has authored or co-authored more than 60 refereed publications in the areas of his research and has received four best-paper awards. He holds a BSc degree in mechanical engineering from the University of Kerala, India, and MS and PhD degrees in mechanical engineering from the University of Kentucky.

Ovadia Shoham is F.M. Stevenson Distinguished Presidential Chair Professor of Petroleum Engineering at the University of Tulsa. Since 1994, Shoham has directed TUSTP, conducting research on compact separators. He has authored or co-authored more than 90 publications in the areas of multiphase flow, multiphase separation, and production operations. He holds BS and MS degrees in chemical engineering from the University of Houston and the Technion in Israel, respectively, and a PhD degree in mechanical engineering from Tel Aviv University. Recently, Shoham published the book *Mechanistic Modeling of Gas-Liquid Flow* for SPE. Shoham is a recipient of the 2003 SPE Production and Operations Award.