# Hybrid Approach by Use of Linear Analogs for Gas-Network Simulation With Multiple Components

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#### Summary

The generalized Newton-Raphson method is routinely deployed in industrial and academic applications to solve complex systems of highly nonlinear equations. Prime candidates for this solution methodology are complex natural-gas transportation networks, of which the nonlinear governing equations can be written in terms of nodal, loop, or nodal/loop formulations to solve for all network pressures and flows. A well-known issue of the Newton-Raphson iterative methodology is its hapless divergence characteristics when poorly initialized or when flow loops are poorly defined in loop or nodal/loop formulations. In this study, a method of linear analogs is discussed, which eliminates the need for user-prescribed flow or pressure initializations or loop definitions in the solution of the highly nonlinear gas-network governing equations. A comprehensive solution strategy based on analog transformations is presented for the analysis of a gas-pipeline network system comprising not only pipes, but also common nonpipe network elements such as compressors and pressure-dependent gas supplies (e.g., wellheads). The proposed approach retains advantages of Newtonnodal formulations, while removing the need for initial guesses, Jacobian formulations, and calculation of derivatives. Case studies are presented to showcase the straightforward and reliable nature of the methodology when applied to the solution of a steady-state gas-network system analysis with pipeline, compressor, and wellhead components.

# Introduction

Natural-gas network systems usually comprise compressors, wells, and several other surface components other than pipelines (Ayala 2013; Larock et al. 2000; Kumar 1987; Osiadacz 1987). Compressor stations are one of the most-important and -common elements in a natural-gas pipeline system. Compressor stations supply the energy to transport gas from supply points to final destination, while overcoming frictional losses in all transmission pipelines. In long-distance transportation, available pressure at supply points may not be sufficient to transport the gas from one location to the final destination. Hence, compressor installation becomes necessary to boost pressure within the network during transportation. A number of important variables are associated with compressor performance, including the amount of gas flow, gas properties, suction and discharge temperatures, and compression ratio (Ayala 2013; Osiadacz 1987). Compression ratio is a cardinal parameter in determining the horsepower required to compress a certain volume of gas and also the discharge temperature of gas exiting the compressor. Optimum locations and pressures at which compressor stations operate could then be identified and analyzed through a network-simulation study. Wellheads, on the other hand, serve as

the source of natural gas entering the natural-gas network. Production rate at wellheads is dependent on wellhead pressure, wellhead shut-in pressure, and wellhead configuration or tubing conductivity, which is defined by the combined effect of the reservoirproductivity index and tubing performance. Wellhead shut-in pressure is usually predefined in the system because it represents reservoir pressure at shut-in conditions adjusted by hydrostatics. With an integrated network-simulation approach, the engineer can assess the effect of well performance and wellhead-pressure specification on network deliverability.

Fig. 1 shows a schematic of a natural-gas network with pipelines, compressor, and wellheads. Modeling and understanding the behavior of such a network system is not a matter of studying the performance of a single constituent component; rather, one must undertake the integrated study of the consequences of the interconnectivity of every component of the system. The simultaneous solution of the resulting set of highly nonlinear equations enables natural-gas-network simulation to predict the behavior of highly integrated networks for a number of possible operating conditions. These predictions are used routinely to make design and operational decisions that impact a network system, taking into account the consequences of interconnectivity and interdependence among all elements within the system. As discussed in detail by Ayala (2013), natural-gas-network simulation entails the calculations of the flow capacity of each network segment and the pressure at each nodal junction. This can be accomplished by, for example, making network flows the primary unknowns of the problem (i.e., the q-formulation or nodal/loop formulation) or by making nodal pressures the primary unknowns (i.e., the *p*-formulation or nodal formulation). The resulting set of governing equations must then be solved simultaneously in terms of the desired target unknowns. A number of network equation-solution protocols have been proposed throughout the decades-including the Hardy-Cross method (Cross 1932) and the linear-theory method (Wood and Charles 1972), but current practice relies heavily on the implementation of the multivariate Newton-Raphson method for the solution of the network equations. Osiadacz (1987), Kumar (1987), and Larock et al. (2000) among others, have presented detailed reviews of these methodologies. A recurrent theme in these methodologies is that their success is heavily dependent on the availability of suitable flow and pressure initial guesses. In this study, we discuss the linear-analog transformation methodology, which eliminates such a significant limitation.

#### The Method of Linear Analogs

**Pipeline Equation for Network Analysis.** Ayala (2013) has shown that to circumvent Newton-Raphson convergence problems and user-defined initialization in pipe networks, an alternative analog system of pipes obeying a simpler linear-pressure analog flow equation of the type

$$q_{Gij} = L_{ij} \cdot \left( p_i - e^{s_{ij}/2} p_j \right)$$
 .....(1a)

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#### Fig. 1—A natural-gas-network schematic.

should be formulated to replace common gas-pipe constitutive equations of the type

$$q_{Gij} = C_{ij} \cdot \sqrt{p_i^2 - e^{s_{ij}} p_j^2}$$
, ....(1b)

where  $C_{ii}$  is the actual pipe conductivity calculated by any of the most-popular gas-flow equations, as described in Appendix A. Gaspipe flows can be written in terms of linear analogs by invoking the L<sub>ij</sub> vs. C<sub>ij</sub> conductivity transformations summarized in Appendix B. As described in Appendix B, when gas-pipe flows are written in terms of Lii-conductivities, all nodal-mass governing equations in the network collapse to straightforward algebraic equations in terms of linear pressures that can be used to simultaneously solve for all nodal pressures in the network by use of any standard method of solution of linear algebraic equations. To illustrate the performance of the methodology, the gas-distribution grid in the Mexico Valley in Fig. 2, as presented by Montoya-O et al. (2000) and Martinez-Romero et al. (2002), is studied. The grid connects 22 cities, and there are 25 pipes connecting these city stations. Pipe specifications are reported by the authors, and elevation changes are deemed unimportant by them. Natural gas is supplied at Venta de Carpio (Node 1) at a pressure of 24.61 bar (356.94 psia), with a gas specific gravity of 0.65 and an average flow temperature of 345°R. Pipes are assumed to be operating at an efficiency of 0.80, with an average gas compressibility of 0.98. The model is run on the basis of the Panhandle-B gas-flow equation, as implemented by the authors. Figs. 3 and 4 demonstrate that when the linear-analog methodology is implemented, pressure and flow-rate estimations, respectively, converge steadily and that convergence behavior remains smooth and steady as the number of iterations increases. Note that no Newton-Raphson iterations are implemented, no ini-



Fig. 2—Network topology of Mexico Valley network—Case Study 1.



Fig. 3—Nodal pressures (p<sub>i</sub>) vs. number of iterations (k)—Mexico Valley network.

tial guesses for flow or pressure were necessary at any point, and the only initialization required is that of linear conductivities for which we used  $L_{ij}=2C_{ij}$ , while any other multiplier could be used because  $L_{ij}>C_{ij}$  (see Appendix B). Final converged values of the Mexico Valley network are summarized in **Table 1**, which compares favorably with all reported nodal-pressure data in Martinez-Romero et al. (2002; see their Table 8, with pressures converted from bar to psia). Maximum deviation errors are found at approximately 1%.

**Compressor Equation Analog for Network Analysis.** In this study, we extend the linear-analog methodology to additional network components, such as compressors and wellheads, not originally considered by Ayala (2013). Compressors are key components in any gas-network system because they supply the energy

required to transport gas from one end to another. The amount of energy contributed to the gas by the compressor is dependent on the gas pressure and flow rate, as shown in Appendix C. By use of the analog transform described in Appendix C, the compressor equation can be incorporated in a straightforward manner into the linear-analog analysis by prescribing a desired compression ratio.

The proposed methodology is illustrated with Case Study 2 (which solves the gas topology in Fig. 1), and reproduced in **Fig. 5** with additional pipe-elevation information and given supplies and demands. The network is analyzed with the generalized gas-flow equation coupled with American Gas Association (AGA) fully turbulent friction-factor calculations (Appendix A). An average flowing temperature of 75°F and an average compressibility factor of 0.90 are assumed for the entire system for illustration purposes; however, it should be clear that the methodology would remain un-



Fig. 4—Pipe flow rate  $(q_{Gii})$  vs. number of iterations (k)—Mexico Valley network.

TABLE 1—NODAL NETWORK PREDICTIONS FOR MEXICO VALLEY (CASE STUDY 1)										
City	Node	Supply (–) / Demand (+) (MMscf/D)	Pressure Reported by Martinez- Romero et al. (2002) (psia)	Calculated Pressure Using Linear-Analog Model (psia)	Deviation Error (%)					
Venta de Carpio	1	-258.58	356.94	356.94	0.00					
Tultitlán	2	11.79	345.77	346.33	0.16					
Lechería	3	14.04	345.77	346.24	0.14					
Yets	4	0.00	346.49	347.00	0.15					
Barrientos	5	0.00	345.77	346.31	0.16					
Anahuac	6	18.96	340.26	340.42	0.05					
Romana	7	15.79	339.39	339.48	0.03					
Comunidad	8	9.58	339.24	339.37	0.04					
Río	9	0.00	339.24	339.36	0.03					
San Juanico	10	17.46	344.32	344.36	0.01					
Cerro Gordo	11	13.92	351.28	351.28	0.00					
Tulpetlac	12	6.29	351.86	351.88	0.01					
Sosa Texcoco	13	27.75	353.31	353.27	-0.01					
Vallejo	14	16.92	338.66	338.80	0.04					
18 de Marzo	15	55.63	289.49	286.58	-1.01					
Camarones	16	9.17	289.35	286.45	-1.00					
Anahuac 78	17	10.33	288.77	285.86	-1.01					
Anahuac 80	18	14.46	288.63	285.77	-0.99					
San Pedro de los Pinos	19	14.25	288.92	286.18	-0.95					
Соара	20	2.25	338.66	338.61	-0.01					
Pinter	21	0.00	352.15	352.24	0.03					
Belén de las Flores	22	0.00	288.92	286.18	-0.95					



Fig. 5—Network topology of Case Study 2.



Fig. 6—Pressure ratio (r<sub>ii</sub>) vs. number of iterations (k)—Case Study 2.

changed if each pipe were to be considered to operate at different average temperatures and if compressibility factors were calculated in terms of standard natural-gas correlations. In this case study, the network handles a gas with a specific gravity of 0.69, and all pipes are assumed to be carbon steel (e=0.0018 in.), horizontal, 30 miles long, and nominal pipe size (NPS) 4 schedule (Sch) 40, except for Pipes (1,2), (2,3), (1,4), and (4,7), which are NPS 6 Sch 40. The pressure specification is given at Node 9, and is set at 130 psia. A compressor is located between Node 5 and Node 6, and Pipe (4,5)and Pipe (6,7) are both 15 miles in length. The compressor is located at an elevation of 400 ft with respect to the datum at Node 11 and is prescribed to deliver a pressure boost of  $r_{c56}=2.5$  while in operation. Suction temperature and the average compressibility of the gas at the compressor are assumed to be 75°F and 0.90, respectively, with a polytropic exponent of 1.40 and a compressor efficiency of 0.90. The compressor is operating at a target compression

ratio of 2.50 with a single stage ( $n_{st}$ =1). Node 11 is again specified at a pressure of 130 psia.

**Figs. 6 through 8** present performance results for the proposed linear analog when compressors are part of the network. Appendix C describes the type of system of algebraic equations that results for Case Study 2. During the non-Newton-Raphson iterative process, pressure ratios (Fig. 6) and pressure and flow-rate estimations are seen to steadily and smoothly converge (Figs. 7 and 8, respectively). For this case study, an  $L_{ij}=C_{ij}$  conductivity initialization was used to illustrate its effects. It should be noted that the use of  $L_{ij}=C_{ij}$  leads to early overestimations of pressures and flow rates because pipe linear conductivities are being underestimated significantly (given that  $L_{ij}>C_{ij}$ ; see Appendix A). Nevertheless, it is shown that the methodology is able to adjust them accordingly as iterations continue. Final converged values, or the actual network solution, are displayed in **Fig. 9**, where the compressor horsepower



Fig. 7—Nodal pressures (p<sub>i</sub>) vs. number of iterations (k)—Case Study 2.



Fig. 8—Pipe flow rate  $(qG_{ii})$  vs. number of iterations (*k*)—Case Study 2.



Fig. 9—Fully converged natural-gas-network distribution—Case Study 2.



Fig. 10—Network topology of Case Study 3.

(HP) is estimated to be 149.60. No Newton-Raphson iterations are used at any point.

Wellhead Equation Analog for Network Analysis. Appendix D presents the necessary analog manipulation required to incorporate gas supplies (wellheads) into gas-network analysis by use of

linear analogs. To illustrate the performance of the methodology, Case Study 3 considers the same network as Case Study 2, but adds two pressure-dependent supplies (wellheads) at Nodes 1 and 9, as shown in **Fig. 10.** The compressor station remains located between Nodes 5 and 6, operating at the target compression ratio of 2.5. Demands are slightly higher at Nodes 3, 7, and 8 compared with the



Fig. 11—Pressure ratio (*r<sub>ij</sub>*) vs. number of iterations (*k*)—Case Study 3.



Fig. 12—Nodal pressures ( $p_i$ ) vs. number of iterations (k)—Case Study 3.

previous case to accommodate for the additional supply emerging from the new wells. Demand at Node 11 is set to be variable and dependent on the specified pressure because supply is now a function of the wellhead nodal pressures. Specified pressure of 130 psia remains at Node 11. Well 1 and Well 9 are assumed to have well conductivities  $C_{w1}$  of 2.0 Mscf/D/psi<sup>1.3</sup> and  $C_{w9}$  of 2.0 MMscf/D/ psi<sup>1.2</sup>, and both of the wells are operating with a shut-in pressure of 1,000 psia. The well-flow exponents are assumed to be 0.65 for the well at Node 1 and 0.60 for the well at Node 9. Appendix D also illustrates the resulting linear system of algebraic equations when the analog methodology is used for this case study.

**Figs. 11 through 14** display the performance of the linear-analog methodology for Case Study 3. As previously seen, pressure ratios are able to steadily converge (Fig. 11) despite the increased nonlinearity in the system. Pressure and flow-rate estimations converge steadily and progressively, as highlighted in Figs. 12 and 13, respectively. Convergence behavior remains smooth and steady. Note

that the flow rates for Pipe (4,9) and Pipe (7,10) start flowing in the opposite direction from the final direction by use of the  $L_{ij}=C_{ij}$ conductivity initialization, with successive iterations able to correct it. Final converged values, or the actual network solution, are displayed in Fig. 14. The compressor is determined to be operating at 201.81 HP, and Wells 1 and 9 are determined to be producing at 13.07 and 6.16 MMscf/D, with wellhead flowing pressures of 509.25 and 589.17 psia, respectively. It is seen that the analog methodology is able to keep its simplicity and reliable nature despite the inclusion of additional components in the pipeline-network system, which increased the complexity and nonlinearity of the simulation significantly. No Newton-Raphson iterations were required for convergence.

# **Concluding Remarks**

A linear-analog method has been presented that is able to solve significantly nonlinear gas-load flow problems for network sys-



Fig. 13—Pipe flow rate (q<sub>Gij</sub>) vs. number of iterations (k)—Case Study 3.



Fig. 14—Fully converged natural-gas-network distribution—Case Study 3.

tems composed of pipes, compressors, and wellhead supplies. The method is novel and successfully preserves some advantages of the Newton-nodal method—namely, solely relying on nodal equations and not needing loop-path identification-while at the same time eliminating the need for Jacobian formulations, for calculation of derivatives, and, quite importantly, for the availability of good initial values for flow rate and pressures. The drawback of the proposed analog method is that its convergence would be nonquadratic and more analog iterations would always be required for final convergence once the iterative solution became close to the actual solution. Such linear-analog iterations would be less expensive and more straightforward to execute than any Newton-Raphson iterations; however, Newton-Raphson quadratic-convergence speed would not be able to be outmatched once the solution became close to the actual solution. The implementation of a hybrid approach emerges as the best overall solution strategy and would be recommended by this study. In this hybrid approach, the proposed linear-analog method should be implemented first to inexpensively and reliably advance the network solution (without the need for pressure and flow guesses) up to a point at which full advantage could be taken of the quadratic convergence of the nodal Newton-Raphson protocol. If the user decides to fully circumvent the need to formulate Jacobian and associated derivative calculations, the method of linear analogs would always provide the choice of fully completing the solution task-unaided, but at the expense of additional iterations.

#### Nomenclature

- $C_{ij}$  = actual pipe conductivity for the generalized gas-flow equation,  $L^{4/m^{-1}/t}$
- $C_{C_{::}} = \text{compressor constant}$
- $\tilde{C}_R^{IJ}$  = reservoir-rock and -fluid-properties conductivity,  $L^{4/m^{-1}/t}$
- $C_w$  = well conductivity, L<sup>4</sup>/m<sup>-1</sup>/t
- d = pipe internal diameter, L
- D = fluid demand at a node in a pipe network, L<sup>3</sup>t<sup>-1</sup>
- e = pipe roughness, L
- $f_F$  = Fanning friction factor [-]
- $F_D = AGA \text{ drag factors } [-]$
- g = acceleration of gravity, Lt<sup>-2</sup>
- $g_c = \text{mass/force unit conversion constant, m/LF^{-1}t^{-2},$ where  $F=\text{m/Lt}^{-2}$ , 32.174 lbm/ft (lbf^{-1} \cdot \text{s}^{-2}) in Imperial units; 1 kg/mN^{-1}\text{s}^{-2} in SI
- *H*, h = elevation with respect to datum, L
  - k = iteration number
  - $k_c = \text{compressor constant}$
  - $\mathbf{K}$  = network-characteristic matrix
  - L = pipe length, L
  - $L_e =$  pipe equivalent length, L
  - $L_{ii}$  = linear-analog pipe conductivity, L<sup>4</sup>/m<sup>-1</sup>/t
  - m = diameter exponent[-]
  - n =flow exponent [-]
  - $n_p = \text{polytropic exponent}[-]$

 $n_{st}$  = number of compression stages [-]

 $O_i$  = summation of off-diagonal entries in the *i*th row for the linear-analog method,  $L^4/m^{-1}/t$ 

$$p = \text{pressure}, \text{m/L}^{-1}\text{t}^{-2} \text{ or } \text{FL}^{-2}, \text{ where } F = \text{m/L}\text{t}^{-2}$$

 $\mathbf{P}$  = network pressure vector

- $q_{Gij}, q_{Gsc} = \text{gas-flow rate at standard conditions for Pipe } (i,j),$  $L^{3}t^{-1}$ 
  - $r_c = \text{compression ratio} [-]$
  - $r_{ij}$  = pressure ratio for the linear-pressure analog method
- $\begin{aligned} r_w &= \text{well-pressure compression ratio } [-] \\ R &= \text{universal gas constant, } m/L^2 T^{-1} t^{-2} n^{-1}, \\ 10.7315 \text{ psia} \cdot \text{ft}^3 (\text{lbm/mol}^{-1} \text{R}^{-1} \text{ in English units,} \\ \text{or } 8.314 \text{ m}^3/\text{Pa} \cdot \text{K}^{-1}\text{g/mol}^{-1} \text{ in SI}) \\ R_e &= \text{Reynolds number } [-] \\ s, s_{ij} &= \text{pipe-elevation parameter } [-] \\ S &= \text{network supply/consumption vector} \\ SG_G &= \text{gas specific gravity } [-] \\ T_{av} &= \text{average absolute temperature, T} \\ T_{ij} &= \text{analog conductivity transform } [-] \\ T_w &= \text{analog well-conductivity transform, } (m/L^{-1}t^{-2})^{2n-1} \\ Z_{av} &= \text{average fluid-compressibility factor } [-] \\ \gamma &= \text{ratio of specific heats} \end{aligned}$ 
  - $\eta$  = compressor efficiency
- $\kappa_{W}, \kappa_{PA},$ 
  - $\kappa_{PB}$  = unit-dependent constants for the gas friction-factor equations (see Appendix A table)
    - $\mu$  = fluid dynamic viscosity, m/L<sup>-1</sup>t<sup>-1</sup>
    - $\sigma$  = unit-dependent constant for the generalized gas-flow equation, T/t<sup>2</sup>L<sup>-2</sup>

#### Subscripts

- av = average
- c = compressor
- G, g = gas
  - i = inlet (upstream)
  - j = exit (downstream)
  - p = polytropic
- R = reservoirshut = shut-in
  - sc = standard conditions (60°F or 520°R and 14.696 psi inEnglish units; 288.71 K and 101.325 kPa in SI)
  - w = well
- wh = wellhead

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## Appendix A—Specialized Gas-Flow Equations

Following Ayala (2013), **Table A-1** presents a summary of the most common pipe-flow equations routinely used for gas-network analysis and their built-in friction-factor calculation assumption. Please note that the appropriate form of the Fanning friction factor must be used in the implementation of the analog methodology, which relies on the generalized gas-flow-equation form.

#### Appendix B—Linear-Analog Model for Pipe Flow

As shown in Table A-1, all gas-pipe constitutive equations can be written in the following general form:

$$q_{Gij} = C_{ij} \cdot \sqrt{p_i^2 - e^{s_{ij}} p_j^2}$$
....(B-1)

The linear-analog model postulates that an alternative analog equation can be used instead:

$$q_{Gij} = L_{ij} \cdot \left( p_i - e^{s_{ij}/2} p_j \right)$$
, ....(B-2)

where  $L_{ij}$  is the conductivity of the "linear analog" pipe obeying Eq. B-2 and  $C_{ij}$  is the actual pipe conductivity conforming to the generalized gas flow in Eq. B-1. Both conductivities are related through the expression

$$L_{ij} = T_{ij} \cdot C_{ij} , \qquad (B-3)$$

where  $T_{ij}$  is the analog conductivity transform. To derive the proper conductivity transformation  $(T_{ij})$ , Eqs. B-1 and B-2 are equated to yield

$$L_{ij} \cdot \left( p_i - e^{s_{ij}/2} p_j \right) = C_{ij} \cdot \sqrt{p_i^2 - e^{s_{ij}} p_j^2} , \qquad (B-4)$$

which can be rewritten in terms of the pressure ratios  $\left(r_{ij} = \frac{p_i}{p_j}\right)$ 

$$L_{ij}^{2} \cdot (r_{ij} - 1) = C_{ij}^{2} \cdot (r_{ij} + 1) , \qquad (B-5)$$

leading to the  $T_{ij}$  transformations presented in **Table B-1.** When gas-pipe flows are written in terms of  $L_{ij}$  conductivities, all nodalmass governing equations in the network collapse to straightforward algebraic equations in terms of linear pressures. This system of algebraic equations can be solved simultaneously for all nodal pressures in the network with any standard method of solution of linear algebraic equations. It should be noted that  $T_{ij} > 1$  always (see Table B-1), leading to  $L_{ij} > C_{ij}$ . Therefore, to obtain a first nodalpressure solution, the  $L_{ij} = C_{ij}$  (or  $L_{ij} = 2C_{ij}$  because  $L_{ij} > C_{ij}$  always) conductivity initialization is used. Once a nodal-pressure solution is available, pressure ratios  $(r_{ij})$  are readily available and linearanalog conductivities can be updated accordingly. Successive



iterations will follow until calculated nodal pressures do not sig- where the constant  $k_c$  can be defined as nificantly change within a prescribed tolerance.

#### Appendix C—Linear-Analog Model for Compressors

Total HP represents the energy per unit time required by the compressor to achieve the desired rise in pressure. The relationship between HP requirements, compression ratio, fluid properties, compressor efficiencies, and the polytrophic coefficient is given by a direct application of the first law of thermodynamics, which yields

$$HP = 0.0857 \frac{n_{st} \cdot n_p}{n_p - 1} q_{G_{ij}} T_i Z_{av} \left(\frac{1}{\eta}\right) \left[ \left(\frac{p_j}{p_i}\right)^{\frac{n_p - 1}{n_{st} \cdot n_p}} - 1 \right].$$
(C-1)

Rearranging the compressor equation into a short-hand equation, one readily obtains

$$q_{G_{ij}} = \frac{HP}{k_c \left[ \left( r_{c_{ij}} \right)^{\frac{n_p - 1}{n_s \cdot n_p}} - 1 \right]}, \dots (C-2)$$

$$k_c = 0.0857 \left( \frac{n_{st} \cdot n_p}{n_p - 1} \right) T_i \left( Z_{av} \right) \left( \frac{1}{\eta} \right).$$
(C-3)

The total compressor ratio for a compression station is calculated as the ratio of its final compressor-discharge pressure to its entry suction pressure:

If the target compression ratio  $(r_{cij})$  is prescribed for the compressor, one can write

$$q_{G_{ii}} = C_{cij} \cdot HP, \qquad (C-5)$$

where the compressor constant is given as

$$C_{cij} = \frac{1}{k_c \left[ \left( r_{cij} \right)^{\frac{n_p - 1}{n_{ar} \cdot n_p}} - 1 \right]}$$
 [MMscf/D/HP]. ....(C-6)

# TABLE B-1—SUMMARY OF LINEAR-PRESSURE ANALOG CONSTITUTIVE EQUATIONS $(L_{ij} = T_{ij} \cdot C_{ij})$

Network Type	Linear-Pressure Analog Constitutive Equation	$T_{ij}$ , Analog Conductivity Transform
Horizontal pipes	$q_{Gij} = L_{ij} \left( \boldsymbol{p}_i - \boldsymbol{p}_j \right)$	$T_{ij} = \sqrt{1 + rac{2}{r_{ij} - 1}}$ , where $r_{ij} = rac{p_i}{p_j}$
Inclined pipes	$\boldsymbol{q}_{Gij} = \boldsymbol{L}_{ij} \left( \boldsymbol{p}_i - \boldsymbol{e}^{\frac{s_{ij}}{2}} \boldsymbol{p}_j \right)$	$T_{ij} = \sqrt{1 + rac{2}{r_{ij} - 1}}$ , where $r_{ij} = rac{p_i}{rac{s_{ij}}{e^{rac{2}{2}} p_j}}$

The compressor equation may then be incorporated into the gas-network system by predefining the compressor desired-totalcompression ratio, which results in the determination of the HP required for the compressor to be solved for as an unknown within the system of equations.

The implementation of the linear-analog methodology always leads to a linear system of algebraic equations in terms of linear nodal pressures, which, in compact notation, can be written as:

K P=S, .....(C-7)

where K is the network-characteristic matrix, P is the networkpressure vector, and S is the system supply/consumption vector. For a system of pipes only, the diagonal entries in the K matrix represent the summation of all off-diagonal entries for each row (i.e., all the linear conductivities of pipes connected to that node). For this type of system, such as the one presented in Case Study 1, matrix K would be banded, symmetric, diagonally dominant, and positive definite after all specified pressure terms are moved to the supply/consumption vector S. However, the application of the linear-pressure analog constitutive equations for elevated pipes coupled with the proposed compressor equation generates the linear system of algebraic equations for Case Study 2 shown in Fig. C-1, which in compact notation is expressed as shown in Eq. C-7. In this case, the P pressure vector is slightly different from the case in which only pipes are modeled (Case Study 1) because the HP also becomes one of the unknowns. For a pipeonly simulation, the characteristic matrix diagonal entries  $O_i$  are just the summation of all off-diagonal entries. For Nodes 5 and 6 in Case Study 2, where the compressor is found,  $O_5 = e^{s_{45}/2} \cdot L_{45}$ and  $O_6 = -C_{C56}$ . All pipe conductivities ( $L_{ij}$ ) remain in MMscf/D/ psi and the compressor constant ( $C_{Cii}$ ) is in MMscf/D/HP in this example. The resulting characteristic matrix K remains banded, but no longer symmetric, as would be the case for a pipe-only network simulation.

#### Appendix D—Linear-Analog Model for Well Supplies

Wells are primarily sources in a natural-gas network. Wells are assumed to be producing from a defined shut-in pressure (i.e., reservoir pressure at shut-in conditions adjusted by hydrostatics), and the flow rate is ultimately dependent on prevailing wellhead pressures. One of the most popular approaches to modeling well deliverability is the use of the classical backpressure equation proposed by Rawlins and Schellhardt (1935). This equation, written at reservoir conditions, is

$$q_{wGi} = C_R \cdot \left(p_R^2 - p_{wf}^2\right)^n$$
, for 0.5<*n*<1, .....(D-1)

where  $C_R$  is the reservoir productivity index. The productivity index is typically obtained from well-testing data or isochronal testing of the well. This equation may also be rewritten at surface (wellhead) conditions by use of the approximation

$$q_{wGi} = C_w \cdot \left(p_{\text{shut}}^2 - p_{wh}^2\right)^n$$
, for 0.5<*n*<1, .....(D-2)

where  $C_w$  is the well conductivity. Please note that  $C_w$  essentially captures or integrates the effects of the reservoir productivity index and tubing performance by use of outflow/inflow nodal analysis. This constitutive relationship retains a form identical to that of the pipeline-gas-flow equation, and, hence, a similar analog transformation could be applied to linearize the wellhead equation. The backpressure equation is expressed in a manner similar to that of the generalized pipe equation, while the coefficients *n* and  $C_w$  vary for different reservoir and tubing properties for the backpressure equation. The linear-analog equation for any wellhead in the network system is then given by

$$q_{wGi} = L_w \cdot (p_{\text{shut}} - p_{wh}).$$
(D-3)

Linear-pressure analog conductivities for a wellhead are again computed as a function of actual well conductivities according to the transformation rule

$-O_1$	$e^{s_{12}/2}L_{12}$	0	$e^{s_{14}/2}L_{14}$	0	0	0	0	0	0	0	$\begin{bmatrix} p_1 \end{bmatrix}$	] [	-16
$L_{12}$	$-O_{2}$	$e^{s_{23}/2}L_{23}$	0	0	0	$e^{s_{27}/2}L_{27}$	0	0	0	0	<i>p</i> <sub>2</sub>		2
0	$L_{23}$	$-O_{3}$	0	0	0	0	$e^{s_{38}/2}L_{38}$	0	0	0	$p_3$		2
$L_{14}$	0	0	$-O_{4}$	$e^{s_{45}/2}L_{45}$	0	0	0	$e^{s_{49}/2}L_{49}$	0	0	$p_4$		3
0	0	0	$L_{45}$	$-O_{5}$	$-C_{C56}$	0	0	0	0	0	$p_5$		0
0	0	0	0	$-r_{56}L_{67}$	$-O_{6}$	$e^{s_{67}/2}L_{67}$	0	0	0	0	HP	=	0
0	$L_{27}$	0	0	$r_{56}L_{67}$	0	$-O_{7}$	$e^{s_{78}/2}L_{78}$	0	$e^{s_{710}/2}L_{710}$	0	<i>p</i> <sub>7</sub>		1
0	0	$L_{38}$	0	0	0	$L_{78}$	$-O_8$	0	0	$e^{s_{811}/2}L_{811}$	$p_8$		2
0	0	0	$L_{49}$	0	0	0	0	$-O_{9}$	$e^{s_{910}/2}L_{910}$	0	$p_9$		2
0	0	0	0	0	0	L <sub>710</sub>	0	$L_{910}$	$-O_{10}$	$e^{s_{1011}/2}L_{1011}$	$p_{10}$		2
0	0	0	0	0	0	0	0	0	0	1	$p_{11}$		130

Fig. C-1—Linear system of algebraic equations for Case Study 2.



Fig. D-1—Analog well-conductivity transform ( $T_w$ ) as a function of well-pressure ratio ( $r_w$ );  $P_{shut}$ =100 psia.

$$L_w = T_w \cdot C_w , \qquad (D-4)$$

where  $L_w$  is the wellhead conductivity in the linear-pressure analog model, which conforms to the linear equation (Eq. D-3), and  $C_w$  is the actual well conductivity conforming to the wellhead equation (Eq. D-2). The analog well-conductivity transform  $T_w$  then becomes a function of the well-flow exponent (*n*) and well shut-in pressure ( $p_{shut}$ ), as shown in Eq. D-5:

$$T_{w} = \left(1 - \frac{1}{r_{w}}\right)^{n-1} \cdot \left(1 + \frac{1}{r_{w}}\right)^{n} \cdot p_{\text{shut}}^{2n-1} , \qquad (D-5)$$

where the wellhead pressure ratio  $r_w$  is given by the ratio of shut-in pressure ( $p_{shut}$ ) to wellhead pressure ( $p_{wh}$ ), as shown by

$$r_w = \frac{p_{\text{shut}}}{p_{wh}} . \tag{D-6}$$

**Fig. D-1** depicts the dependency of the analog-well conductivity transform for a range of flow exponents. Because  $T_w > 1$ , as shown in the figure, resulting linear-analog conductivities  $(L_w)$  have larger values than actual well conductivities  $(C_w)$  (i.e., linear-analog wells are more "productive" than their gas counterparts in terms of the absolute values of these conductivities). Similarly, because  $r_w$  needed in Eq. D-5 is not available until iterations have started,  $L_w$  is initialized in this study by use of the approximation  $T_w = p_{\text{shut}}^{2n-1}$ , such that

$$L_w = p_{\text{shut}}^{2n-1} \cdot C_w. \tag{D-7}$$

While updating  $T_w$  values according to Eq. D-5, care must be exercised to enforce that pressure ratios  $(r_w)$  remain always positive, higher than unity, and with values that should not exceed  $r_w = p_{shut}(psia)/14.7$ .

For Case Study 3, for example, the application of the linearpressure analog constitutive equation coupled with compressor and wellhead equations generates the linear system of algebraic equations shown in **Fig. D-2**, which in compact notation is expressed as

The structure of this system of equations is similar to that for the system of Case Study 2 (Appendix C). However, for the network consumption/supply vector **S**, production at Nodes 1 and 9 is essentially dependent on wellbore performance, which is represented by the entry  $-L_{wi} \cdot p_{shut}$ . In addition, for the nodes with wells, values of  $L_w$  are incorporated in the corresponding diagonal entries in the characteristic matrix **K**. For instance,  $O_1 = L_{12} + L_{14} + L_{w1}$  and  $O_9 = L_{49} + L_{910} + L_{w9}$ . The resulting characteristic matrix **K** remains banded and asymmetric.

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$-O_1$	$e^{s_{12}/2}L_{12}$	0	$e^{s_{14}/2}L_{14}$	0	0	0	0	0	0	0 ]	$\begin{bmatrix} p_1 \end{bmatrix}$	$\left[-P_{\text{shut}} \cdot L_{w1}\right]$
$L_{12}$	$-O_{2}$	$e^{s_{23}/2}L_{23}$	0	0	0	$e^{s_{27}/2}L_{27}$	0	0	0	0	$p_2$	2
0	$L_{23}$	$-O_{3}$	0	0	0	0	$e^{s_{38}/2}L_{38}$	0	0	0	$p_3$	2
$L_{14}$	0	0	$-O_4$	$e^{s_{45}/2}L_{45}$	0	0	0	$e^{s_{49}/2}L_{49}$	0	0	$p_4$	3
0	0	0	$L_{45}$	$-O_{5}$	$-C_{C56}$	0	0	0	0	0	<i>p</i> <sub>5</sub>	0
0	0	0	0	$-r_{56}L_{67}$	$-O_{6}$	$e^{s_{67}/2}L_{67}$	0	0	0	0	HP =	= 0
0	$L_{27}$	0	0	$r_{56}L_{67}$	0	$-O_{7}$	$e^{s_{78}/2}L_{78}$	0	$e^{s_{710}/2}L_{710}$	0	<i>p</i> <sub>7</sub>	1
0	0	$L_{38}$	0	0	0	$L_{78}$	$-O_{8}$	0	0	$e^{s_{811}/2}L_{811}$	$p_8$	2
0	0	0	$L_{49}$	0	0	0	0	$-O_{9}$	$e^{s_{910}/2}L_{910}$	0	$p_9$	$-P_{\rm shut} \cdot L_{w9}$
0	0	0	0	0	0	L <sub>710</sub>	0	$L_{910}$	$-O_{10}$	$e^{s_{1011}/2}L_{1011}$	$p_{10}$	2
0	0	0	0	0	0	0	0	0	0	1	$p_{11}$	130

Fig. D-2—Linear system of algebraic equations for Case Study 3.