System-Reliability Analysis by Use of Gaussian Fuzzy Fault Tree: Application in Arctic Oil and Gas Facilities

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Summary

Reliability analysis has various applications in oil- and gas-processing facilities, such as identifying the bottlenecks of the system, quantitative risk assessments, improving system availability and throughput capacity, spare-parts planning, and optimizing maintenance strategies. Reliability performance of a system can be described as a function of operation time and a series of operating conditions. For this purpose, a range of reliability data is required, on the basis of which the reliability function can be modeled. One of the challenges in reliability analysis of Arctic oil and gas facilities is lack of adequate reliability data. The available historical data gathered in normal-climate regions may not be appropriate because they do not include the effects of harsh Arctic operating conditions on equipment performance.

In this study, the expert-judgement process is used as a tool to modify the mean time to failure of the equipment to include the adverse impacts of Arctic climate conditions on equipment performance. However, various sources of bias and uncertainties are involved in expert judgements. Fuzzy set theory is used to deal with such uncertainties and their propagation in both the component- and system-level analyses. For this purpose, a methodology is presented to perform a Gaussian fuzzy fault-tree analysis for system-reliability assessments. This methodology is further illustrated by a case study.

Introduction

Reliability is defined as "the ability of an item to perform a required function under stated conditions for a stated period of time" (*ISO 8402:1994*). The term "ability" can be expressed quantitatively with probability, referring to the chance or likelihood that an item will perform its intended function. The term "stated conditions" emphasizes that an item may perform its intended functions adequately under one set of conditions (operational environment) and quite poorly under another set (Stapelberg 2009).

There are several tools, such as reliability block diagram (Gao et al. 2010), Markov models (Malefaki et al. 2014), fault-tree models (Wang et al. 2013; Yuhua and Datao 2005), and Monte Carlo simulation (Zio et al. 2007), that can be used to describe the system reliability mathematically as a function of the reliability performance of its components (Bauer et al. 2009). Fault-tree analysis (FTA) is a deductive system analysis that has been used extensively in quantitative risk assessments and prediction of system-failure probability in various fields, such as nuclear power plants (Purba 2014), chemical process plants (Wang et al. 2013), oil and gas industry (Yuhua and Datao 2005), and aerospace (Phillips and Diston 2011). An FTA provides a comprehensive and structured approach to estimate system-failure probability, identify and understand key system vulnerabilities, develop accident scenarios, and assess the safety level in facilities. Through an FTA, one postulates that the system itself has failed in a certain way that will be considered as a top event. The occurrence of the top event is further described in terms of the occurrence or nonoccurrence of other intermediate/basic events. Some mathematical methods, such as Boolean algebra, rare-event approximation, and set theory, are then used to estimate the probability of the top event (i.e., system failure) as a function of intermediate/basic events (i.e., component/subsystem failures) (Bauer et al. 2009; Bedford and Cooke 2001; Vesely et al. 1981). Failure probability of a basic event can be estimated by analyzing the life data that can be available from operational or field experiences, maintenance reports, reliability tests, historical data, and handbooks.

Because the oil and gas industry has less experience in Arctic regions compared with normal-climate regions, adequate life data may be sparse. Available data from normal-climate regions may not be suitable for reliability analysis of Arctic oil and gas facilities because of the considerable differences in operating conditions. The harsh operating environment in the Arctic is commonly described as extremely low temperatures, winds, snowdrifts, polar low pressures, atmospheric and sea-spray icing, sea-ice-induced vibrations, seasonal darkness, and poor visibility resulting from fog and snowstorms. Because the reliability performance of Arctic oil and gas facilities is adversely affected by such an environment (Barabadi et al. 2013; Naseri and Barabady 2013), the corresponding reliability assessments must be performed in accordance with the adverse effects of harsh operating conditions on the equipment and operation performance.

Proportional hazard models (Barabadi and Markeset 2011; Gao et al. 2010) and accelerated life models (Barabadi 2014) are applied to include the effects of Arctic operating conditions on component-reliability performance. However, such models rely on an extensive range of detailed data that may not be available, particularly in the Arctic regions. Thus, the expert-judgement process can be applied as an alternative method to cope with this shortcoming and account for such effects. Expert judgements represent the experts' state of knowledge regarding a technical question at the time of response. Such judgements are expressions of opinion that are based on knowledge and experience. Expert judgment is not restricted to the experts' answer, but includes the experts' mental processes of assumptions, definitions, and algorithms, through which the answers are formulated (Ortiz et al. 1991). The concept of expert judgement has been applied in a variety of fields, including nuclear engineering, meteorological research, aerospace, seismic and environmental risk, and risk and safety analysis of oil and gas operations (Clemen and Winkler 1999; Moon and Kang 1999; Purba 2014). In most of these studies, experts are mainly asked to provide a qualitative idea about the frequency of occurrence of an event. Such qualitative words are then converted to fuzzy linguistic variables and, consequently, to fuzzy numbers for further quantitative assessments.

In this study, a methodology is presented for system-reliability assessment on the basis of fuzzy FTA, which is applicable to Arctic oil and gas operations. More specifically, the expert-judgement

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process is used to modify available life data gathered in normal-climate regions to include the effects of Arctic operating conditions on reliability performance of components and systems. Fuzzy set theory is used to deal with the uncertainties involved in expert judgements. For this purpose, the exact values of mean time to failures are combined with the subjective opinions of the experts, which are converted to Gaussian fuzzy numbers. To develop a model for system-reliability analysis and to analyze the corresponding uncertainty propagation, a fuzzified form of FTA is developed. The proposed methodology is illustrated by a case study consisting of a three-phase horizontal separator and its surrounding valves. The remainder of this paper is organized as follows: a short introduction to fuzzy set theory and Gaussian fuzzy numbers is presented. The methodology for performing the Gaussian fuzzy FTA is then described. Conclusions are presented after illustrating the methodology by a case study.

Fuzzy Set Theory

In classical set theory, a set is defined as a collection of objects or elements out of a universal set that share common properties or characteristics. In that regard, an element receives a membership degree of unity, if and only if that element belongs to the set; otherwise the membership degree will be zero (i.e., the element does not belong to the set). For example, considering the classical set concept, if the failure probability of a component before a certain time is 2%, its membership grade is unity and all other failure-probability values have a membership degree of zero because they are not included in the set $F = \{2\%\}$. However, the classical set may reach its limit where the property that distinguishes the members from nonmembers is ambiguous and vague because of, for instance, some uncertainties. While in classical set theory, a sharp, crisp, and unambiguous boundary distinguishes the members and nonmembers for any well-defined set of entities, fuzzy set theory accepts partial memberships. On the basis of fuzzy set theory, which was introduced by Zadeh (1965), it is allowed to have an element that at the same time belongs to a set and does not belong to that set. The degree at which the element belongs to the set is assigned by a membership function (Chen and Pham 2001; Dubois and Prade 1980). The higher the membership grade, the more the element belongs to the set. Considering the aforementioned example, to account for the uncertainties, one may also consider the failure probability values of 1.88 or 2.02% as members of $F = \{2\%\}$, but with a membership degree of less than unity, for instance 0.95.

Expert opinions can be formulated with the fuzzy set theory. The impacts of an Arctic operating environment on equipment reliability performance may vary based on a number of factors, including equipment type, equipment function, equipment location on the platform, and the severity of the weather conditions. In this regard, exact quantification of all such impacts in the form of single-point values is not feasible. Thus, experts prefer to present their opinions by use of a range or quantiles of a distribution to reflect the uncertainties associated with their ideas. Then, fuzzy set theory can be used to combine the expert opinions and their associated uncertainties, and finally, to present the failure probability in fuzzy form.

Mathematically, a fuzzy number $\tilde{X}$ is a convex normalized fuzzy set of the real line $\mathbb{R}$, which is defined as $\tilde{X} = \left\{\left[x, \mu_{\tilde{X}}(x)\right], x \in \mathbb{R}\right\}$, where $\mu_{\tilde{X}}(x) \in [0,1]$ is the membership grade of the element x in $\tilde{X}$. The membership function $\mu_{\tilde{X}}(x)$ is piecewise continuous, and there is exactly one $x_M \in \mathbb{R}$, where $\mu_{\tilde{A}}(x_M) = 1$ (Dubois and Prade 1980). There are various types of fuzzy numbers, some of which are of particular interest because of the specific behavior of their membership functions such as triangular, trapezoidal, Gaussian, exponential, and quadratic. A Gaussian fuzzy number $\tilde{X}$ over a universal set $\mathbb{S}$ is defined as a fuzzy number whose membership function is characterized by a normalized and, in general, asymmetrically parameterized Gaussian function, given as (Hanss 2005)

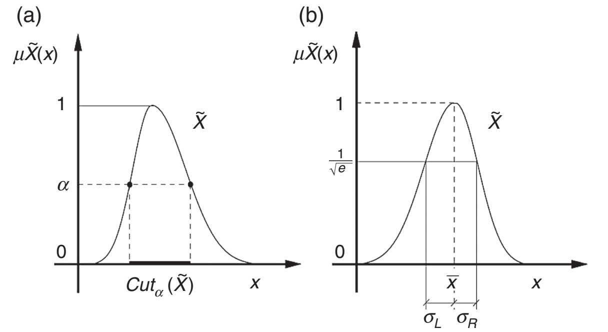

Fig. 1—(a) A typical Gaussian fuzzy number $\tilde{X}$ and (b) its α-cut set.

$$\mu_{\tilde{X}}(x) = \begin{cases} \exp\left[-(x - x_M)^2 / (2\sigma_L^2)\right], \text{ for } x < x_M \\ \exp\left[-(x - x_M)^2 / (2\sigma_R^2)\right], \text{ for } x \geq x_M \end{cases} \quad \text{(1)}$$

Because the element x_M (known as the mean value of $\tilde{X}$) has a membership grade of unity, the fuzzy number $\tilde{X}$ can be considered as the fuzzified form of the crisp number x_M. Fuzzy numbers can be described effectively with the important concept of the α-cut set, which facilitates the fuzzy arithmetic operations. The α-cut set of the fuzzy number $\tilde{X}$ is defined as a crisp set of elements x, with membership grades being greater than or equal to some threshold $\alpha \in (0,1)$. The α-cut set of $\tilde{X}$ is mathematically expressed as $X_\alpha = \left[x \in \mathbb{R} \middle| \mu_{\tilde{X}}(x) \geq \alpha\right]$ (Dubois and Prade 1980). A Gaussian fuzzy number $\tilde{X}$ can also be defined using its crisp α-cut set $X_\alpha = (x_{\alpha L}, x_{\alpha R}), x_{\alpha L} \leq x_{\alpha R}$, where $x_{\alpha L}$ and $x_{\alpha R}$ are given as $x_{\alpha L,R} = \mu_{\tilde{X}}^{-1}(\alpha)$ (Hanss 2005). **Figs. 1a and 1b** show a typical Gaussian fuzzy number and its crisp α-cut set, respectively. The method of determining the α-cut set of a Gaussian fuzzy number is given in Appendix A.

Extension Principle and Fuzzy-Number Arithmetic. By use of the extension principle introduced by Zadeh (1965), the domain and range of an ordinary function from ordinary sets are extended to fuzzy sets. The extension principle provides a general method for extending the crisp or nonfuzzy mathematical concepts to deal with fuzzy numbers and fuzzy functions (Dubois and Prade 1980). This principle can be used when one wants to fuzzify a function or to include the associated uncertainties in the function parameters, and thus evaluate the uncertainty propagation. In this study, the extension principle is used to fuzzify the expert opinions, combine them, and then form the reliability or failure-probability function for the system and its components.

The extension principle is defined as follows (Zadeh 1965): Suppose that G is an ordinary function $G: \mathbb{R}^n \mapsto \mathbb{R}$ that maps an element $(x_1, x_2, \ldots, x_n)$ to the element $y = G(x_1, x_2, \ldots, x_n)$. Additionally, let $\tilde{X}_i \subseteq \mathbb{R}$ be a fuzzy set defined by a membership function $\mu_{\tilde{X}_i}(x_i)$, $x_i \in \mathbb{R}$, $i = 1, 2, \ldots, n$. Then, using the concept of α-cut set, the membership function $\mu_{\tilde{Y}}(y)$, $y \in \mathbb{R}$ of fuzzy set $\tilde{Y} \subseteq \mathbb{R}$ with $\tilde{Y} = G(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n)$ is defined as

$$\begin{aligned} Y_\alpha &= G(X_{1,\alpha}, X_{2,\alpha}, \ldots, X_{n,\alpha}) \\ &= \left[y \in \mathbb{R} \middle| y = G(x_1, x_2, \ldots, x_n), x_1 \in X_{i,\alpha}\right]. \end{aligned} \quad \text{(2)}$$

Gaussian Fuzzy Fault-Tree Analysis (FTA)—Methodology

This study proposes a methodology consisting of two phases. During Phase I, the system-reliability model is developed for the base area, where the life data are available for the system and its components. The operating conditions in the base area are considered normal. During Phase II, the developed model in Phase I is

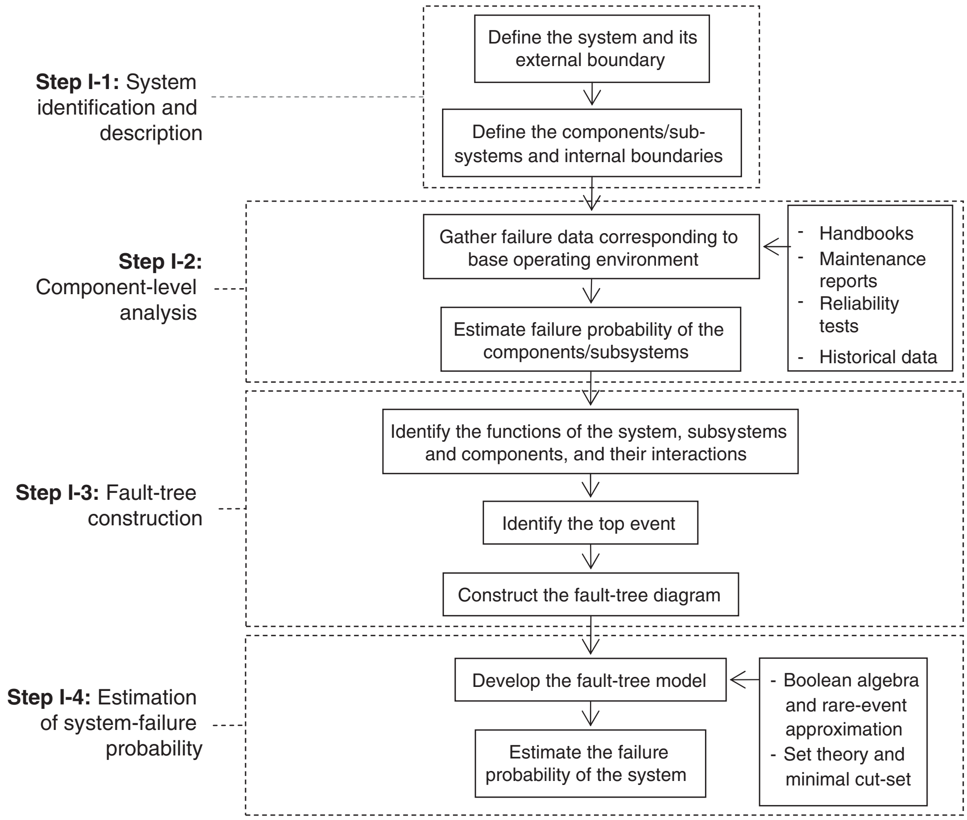

Fig. 2—Procedure for Phase I: system reliability modeling for the base area.

modified by expert judgements and fuzzy set theory to develop the system-reliability model for the target area (i.e., an Arctic region), where the life data are sparse or not available. The aim of this phase is to include the subjective opinions of experts about the potential impacts of the harsh Arctic operating environment on reliability performance of the system and its components.

Phase I: System-Reliability Modeling for the Base Area. The aim of Phase I is to perform an FTA for the base area to estimate the system reliability as a function of each component's reliability or failure probability. For this purpose, a set of steps must be followed, as illustrated by **Fig. 2.**

Step I-1: System Identification and Description. A system is defined as a "set of interrelated elements considered in a defined context as a whole and separated from their environment" (*IEC 60050-151, 2001*). The elements of a system may also be broken down to subsystems and components. Internal boundaries are used to establish a limit of resolution and to determine in how much detail one should study the system. Additionally, to decide what factors could influence the system function, or to determine what aspects of the system performance are of concern, one needs to establish the external boundaries of the system in question (Bedford and Cooke 2001; Vesely et al. 1981).

Step I-2: Component-Level Analysis. The aim of this step is to develop the reliability or failure-probability function for each component. To perform this task, a set of life data is required, to which a theoretical distribution is fitted. Such detailed data can be acquired from handbooks, maintenance reports, reliability tests, and historical data. Let T be a random variable representing the time to failure of a component. The probability that the component fails before time t is called failure probability or unreliability. As per the probability terminology, failure probability $F(t)$ is the same as the cumulative distribution function of the random variable T, which is given by

$$F(t) = P(T \le t) = \int_0^t f(x)\,\mathrm{d}x, \quad (3)$$

where $f(x)$ is the probability-density function of the continuous random variable T, such that $\int_0^{+\infty} f(x)\,\mathrm{d}x = 1$ (Verma et al. 2010). Assuming a one-parameter exponential distribution, the probability-density function can be written as

$$f(t) = \lambda \exp(-\lambda t), \quad (4)$$

where λ is the failure rate of the component. For an exponential probability-density function, failure rate is constant and is defined as the inverse of mean time to failure (MTTF) (i.e., $\lambda = 1/MTTF$). Substituting Eq. 4 into Eq. 3 gives a description for component-failure probability $F(t)$ as

$$F(t) = 1 - \exp\left(-\frac{t}{MTTF}\right). \quad (5)$$

Mathematically, reliability is expressed as the probability that a component fails at a time greater than or equal to a specified time t [i.e., $R(t) = P(T \ge t) = 1 - \mathrm{F}(t)$]. In some contexts, reliability function is known as survival function.

Step I-3: Fault-Tree Construction. A fault-tree model is simply described as a graphic model of the various parallel and sequential combinations of faults that will result in the occurrence of the top event. The faults can be associated with hardware failures, human errors, or any other relevant event that can lead to the top or an intermediate event (Vesely et al. 1981). To develop a fault tree, a number of symbols are generally used to describe events and their combinations. The primary events of a fault tree are those that have not been further developed. Such events can be categorized into basic event, undeveloped event, conditioning

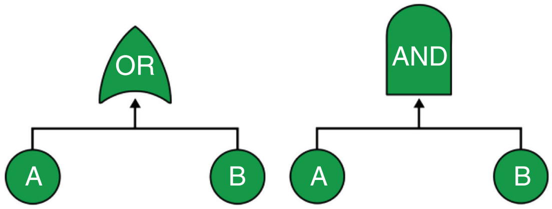

Fig. 3—OR-gate and AND-gate illustration of two basic events *A* and *B*.

event, and external event. A basic or initiating event, represented by a circle, requires no further development and thus signifies the appropriate limit of resolution. In addition to events, a fault tree consists of various logic gates, of which AND-gate and OR-gate are illustrated in **Fig. 3** as two basic types of fault-tree gates. An AND-gate shows that the output fault occurs only if all the input faults occur, while an OR-gate shows that the output fault occurs only if at least one of the input faults occurs (Bedford and Cooke 2001; Vesely et al. 1981).

Step I-4: Estimation of System Failure Probability. There are different methods to estimate the probability of the top event of a fault tree. The rare-event approximation uses the concept of Boolean algebra and its associated rules. To have a more-precise estimation, one can use the simplified Boolean expression of the fault tree to determine the possible minimal cut-sets. A minimal cut-set is the smallest combination of basic events, which, if they all occur, will result in the top-event occurrence; and if one of the failures in the cut set does not occur, then the top event will not occur by that combination (Vesely et al. 1981). Set theory and its associated rules can be further applied to the specified minimal cut-set. The required formulas to estimate the probability of an OR-gate or AND-gate are given by (Verma et al. 2010; Vesely et al. 1981):

$$F(A \text{ and } B) = F(A \cap B) = F(A)F(B) \quad \text{(6)}$$

and

$$F(A \text{ or } B) = F(A \cup B) = F(A) + F(A) - F(A)F(B), \quad \text{(7)}$$

where $F(A)$ and $F(B)$ are failure probabilities of A and B, respectively. One can also describe the AND- and OR-gates by use of the reliability terminology.

FTA is recommended in this study because it is a powerful tool in complex-system analysis, especially where dependent failures or operational loops are present. The other advantage of constructing a fault tree is to determine the minimal cut-sets by applying Boolean algebra concepts. Identifying the minimal cut-sets helps to understand and identify all possible system-failure scenarios. Once the minimal cut-sets are determined, one may perform the system failure or reliability analysis with the reliability block-diagram concept because, in some cases, it may reduce the number of calculations considerably.

Phase II: System-Reliability Modeling for the Target Area. The aim of Phase II is to include the adverse effects of the Arctic operating environment on the system-reliability performance. For this purpose, the FTA performed during Phase I needs to be modified with the subjective opinions of experts that are aggregated with the concept of fuzzy set theory. A number of experts are asked to give their opinions on the degree of decrease in MTTF of the components. Such opinions will be converted to Gaussian fuzzy numbers and will be further combined with appropriate aggregation methods on the basis of the extension principle. More specifically, a set of steps must be followed, as illustrated by **Fig. 4.**

Step II-1: Expert Selection. Expert selection is the first step in the expert-judgment process, and refers to choosing an appropriate number of reliable experts. Various definitions are available for the

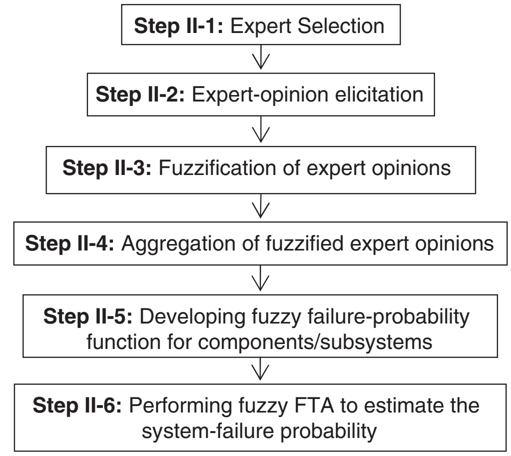

Fig. 4—Procedure for Phase II: system reliability modeling for the target area.

term "expert." An expert can be defined as "a person who has a background in the subject matter at the desired level of detail and who is recognized by his/her peers or those conducting the study as being qualified to solve the questions" (Meyer and Booker 1991). O'Hagan et al. (2006) state simply that "an expert may, in principle, just mean the person whose judgements are to be elicited." Selection of experts is a major issue in expert-judgement studies. This is because the term expert is open to different interpretations because experts are defined by subjective expressions such as "having a desired level of detailed background," "being recognized by their peers," and "being qualified." Additionally, on the one hand, it is advantageous to select a group of experts with wide background, but on the other hand, the analyst may be under pressure to exclude some of the experts who are perceived as being less experienced (Bedford and Cooke 2001).

Step II-2: Expert-Opinion Elicitation. "Elicitation" is defined as the process of obtaining the subjective opinions of experts through specifically designed methods of communication, such as surveys, interviews, group meetings, and questionnaires (Meyer and Booker 1991). Elicitation may be performed in qualitative or quantitative forms. In quantitative form, experts are asked to express their subjective opinions about a parameter in the form of, for instance, a single-point or distribution estimation, an absolute rating, an interval scaling, and a ratio scaling (Cooke 1991; Svenson 1989).

In this study, experts are asked to provide their opinions on the degree of decrease in MTTF of various components operating in the target area as a fraction of the corresponding MTTF in the base area. To include the uncertainty in expert judgements, experts are required to express their opinions in the form of 5, 50, and 95% quantiles (i.e., eliciting the quantiles of a distribution). Uncertainties and biases are important concepts that one needs to take into account while performing the elicitation step. Several studies list three categories of biases that can be introduced to the study at the elicitation step, including structural biases, motivational biases, and cognitive biases (Benson and Nichols 1982; Meyer and Booker 1991; Ortiz et al. 1991; Otway and Winterfeldt 1992).

Structural biases occur when experts are influenced by the way in which a problem is formulated or by the level of detail in the study specified by the analyst. For instance, asking for quantiles of a distribution may lead to different judgements if the distribution parameters were elicited.

Motivational biases occur when the experts may benefit from the results of the study, or when experts express their opinions to please the interviewer or analyst. For example, if the aim of a study is to show and quantify the differences between the equipment-reliability performance in Arctic- and normal-climate regions, experts may bias their opinions by giving a wide distribution on the amount of decrease in equipment MTTF operating in the Arctic, which is in favor of the study goal. Alternatively, some other experts working in the equipment-design field (e.g., manufacturers) may argue that the MTTF of an equipment unit operating in the Arctic is statistically the same as one operating in normal-climate regions.

Cognitive biases are expressed in various ways, including overconfidence, anchoring, and availability. Overconfidence occurs when an expert has a tendency to be more precise about their probability estimates, which may consequently result in presenting distributions that are too tight. Anchoring occurs when an expert anchors to an original estimate that is generally as defensible as possible. Such judgements may be formed on the basis of a known disaster or failure scenario. Availability refers to a cognitive bias in which the frequency of events that are easily imagined or recalled are likely to be overestimated, while more-common-failure scenarios, with less-significant consequences, can be underestimated. These biases can be described for the experts to help them to reduce the level of such biases and their resulting uncertainties. Alternatively, the analyst may present the questions in such a way as to reduce the structural biases. More-detailed discussion regarding the biases and how to deal with them is presented in Meyer and Booker (1991).

Step II-3: Fuzzification of Expert Opinions. Let $\overline{T}_{j,B}$ represent the MTTF of component *j* in the base area. Expert *i* provides his or her subjective opinion on parameter δ, which is the degree of decrease in $\overline{T}_{j,B}$ in the form of [$\delta_{i,L}$, $\delta_{i,M}$, $\delta_{i,R}$]. Parameter δ is given as a fraction of $\overline{T}_{j,B}$ using 5, 50, and 95% quantiles, respectively, given by $\delta_{i,L}$, $\delta_{i,M}$, and $\delta_{i,R}$. In this regard, the MTTF of component *j* in the target area $\overline{T}_{j,T}$ is defined as

$$\overline{T}_{j,T} = (1-\delta)\overline{T}_{j,B}. \qquad (8)$$

However, because there is usually more than one expert involved in the studies, expert opinions need to be combined to form a solution for the analyst or decision maker. Such a solution will be further used to modify the MTTF data gathered in the base area. To combine the elicited expert opinions while including the associated uncertainties, one can describe the quantiles given by each expert in the form of a Gaussian fuzzy number $\tilde{\Delta}_i = \left\{\left[\delta_i, \mu_{\tilde{\Delta}_i}(\delta_i)\right], \delta_i \in (0,1)\right\}$. The membership function $\mu_{\tilde{\Delta}_i}(\delta_i)$ is expressed by use of Eq. 1. The quantiles given by the experts are used to determine the parameters σ_L and σ_R in Eq. 1. The detailed procedure to determine such parameters and to finally fuzzify the quantiles given by the experts is presented in Appendix A.

Step II-4: Aggregation of Fuzzified Expert Opinions. Aggregation of expert opinions refers to the procedure by which the expert judgements are combined by the analyst to provide a basis for the decision maker. Axiom-based approaches are mathematical aggregation methods that are based mainly on the linear and logarithmic opinion-pool principles, of which the former refers to the weighted linear combination and the latter refers to the weighted geometric combination (Bedford and Cooke 2001; Clemen and Winkler 1999; Cooke 1991). Eqs. 9 and 10, respectively, give the weighted linear and geometric combination rules in crisp form:

$$\delta = \sum_{i=1}^{N} w_i \delta_i \qquad (9)$$

and

$$\delta = \prod_{i=1}^{N} \delta_i^{w_i}, \qquad (10)$$

where δ_i is the opinion of Expert *i*, δ is the combined expert opinions in crisp form, *N* is total number of experts, and w_i is the normalized nonnegative weight for Expert *i*. However, to develop a fuzzy relation for these combination rules, the extension principle is applied to already fuzzified expert opinions (see Appendix A).

Various methods are available to assign a weight for each expert, such as assigning equal weights; asking experts to weight themselves; ranking experts in a specific preference, and then assigning weights proportional to ranks; determining weights on the basis of the elicited data; using proper scoring rules; and calibrating experts on the basis of their performance (Cooke 1991). In this study, the equal and experience-based weighting approaches are used.

Step II-5: Developing Fuzzy Failure-Probability Function for Components/Subsystems. Substituting Eq. 8 into Eq. 5 gives the failure-probability function of a component operating in the target area as

$$F(t) = 1 - \exp\left[-\frac{t}{(1-\delta)\overline{T}_{j,B}}\right]. \qquad (11)$$

The fuzzified failure-probability function is in fact an extension of the failure-probability function from ordinary sets into fuzzy sets. In other words, the failure probability of a component until a certain time is no longer a crisp value, but a fuzzy number that includes the uncertainties caused by the vague and complex effects of operating conditions on the component reliability performance (i.e., parameter δ, the quantiles of which are given by experts), and those uncertainties caused by expert judgements. To develop the fuzzified failure-probability function of a component, one needs to use the fuzzified form of aggregated expert opinions (i.e., Step II-3). The detailed description of fuzzy failure probability and its membership function is given in Appendix A.

Step II-6: Performing Fuzzy FTA To Estimate the System-Failure Probability: The fuzzy FTA can be carried out by substituting the fuzzy failure probability of each component into the fuzzy AND-gate and OR-gate. The corresponding membership functions can be determined by applying the extension principle and α-cut set concept (see Appendix A). While the failure probability of the top event is a crisp value in classical FTA, in fuzzy FTA, the failure probability of the top event is a fuzzy number that assigns a membership grade for different values of failure probability.

Case Study

To illustrate the proposed methodology a three-phase, horizontal oil and gas separator system is chosen as a case to estimate its reliability performance under Arctic operating conditions.

Illustration of Phase I. ***Step I-1: System Identification and Description.*** **Fig. 5a** shows a typical three-phase gravity-type horizontal separator. Wellstream enters the separator, where the associated gas and water cut are separated from the oil phase. Gas and water leave the separator vessel through pressure-control and water-dump valves, respectively. The separated oil is then routed to the next-stage separator through the oil-dump valve (Arnold and Stewart 2008). Additionally, according to *API SPEC 12J* (1989), some pressure-relief valves are required to be installed on the vessel. On the basis of these descriptions, the simplified separator system, its components, and its external boundary are shown in Fig. 5b.

Internal boundaries are chosen in such a way that each valve will be considered a single component without further analysis of its internal sections and components, such as actuator, control and monitoring devices, seals, seat rings, and valve body. The internal boundary for the separator is also defined in such a way that the vessel and all of its internal sections, level- and pressure-control devices, and monitoring instruments are considered as a single component.

Step I-2: Component-Level Analysis. To perform the component-level analysis, the mean-time-to-failure (MTTF) data for each component are obtained from the Offshore Reliability Data

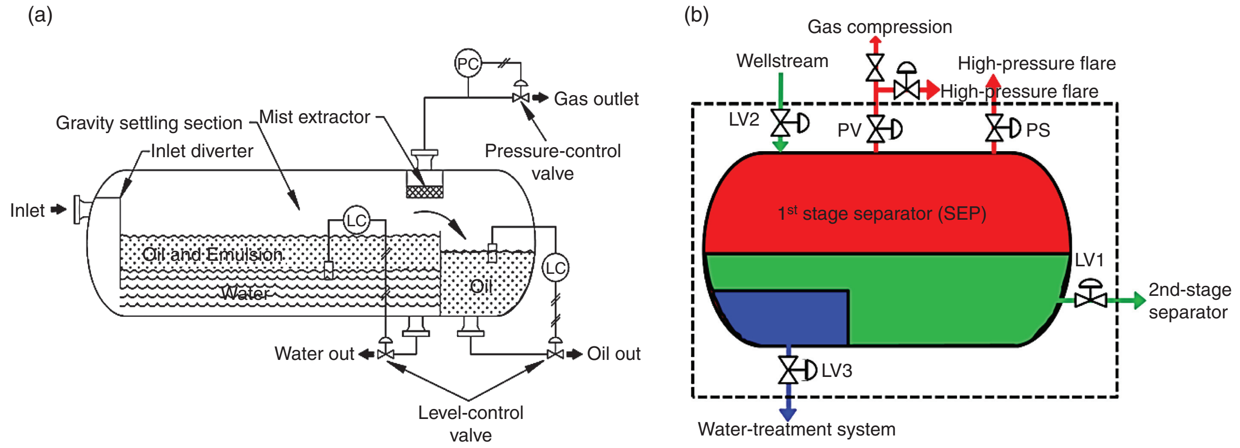

Fig. 5—(a) Schematic of a three-phase separator with interface level control and weir; (b) system external boundary of a separator and its valves.

(OREDA) handbook (OREDA Participants 2009), which includes the failure rate and mean time to repair of a wide range of equipment installed on oil and gas production facilities on the Norwegian continental shelf, except the Barents Sea. There are various types of failure data reported in the OREDA handbook, such as degraded and critical, of which the critical failures "cause immediate and complete loss of an equipment unit's capability of providing its output." The degraded failures "are not critical, but they prevent an equipment unit from providing its output within specifications" (OREDA Participants 2009). Therefore, in this study, the MTTF data are those related to both the critical and degraded failures. However, one can also consider only the critical failures in the analyses. The probability of failure for each component is estimated by use of Eq. 5 at a reference time of t_0 = 2,160 hours (approximately 3 months). Component reliability is also estimated for each component as a function of time, as well as at the reference time. **Table 1** presents the list of components, their identifications, MTTF data, and the reliability and failure probability at the reference time. It is assumed that all level-control valves have identical failure rates. This assumption stands for the pressure-relief and pressure-control valves.

Step I-3: Fault-Tree Construction. To construct the fault-tree diagram, it is assumed that if any of the described valves or separators fail, then the entire separation process fails until the corrective-maintenance tasks restore the failed components to their functioning state. On the basis of this assumption, the fault-tree diagram can be constructed as shown in **Fig. 6.**

Step I-4: Estimation of System-Failure Probability. By applying the concept of minimal cut-sets, the failure-probability function of the system can be modelled as

$$F_{\text{System}} = F\left(S \cup LV1 \cup LV2 \cup LV3 \cup PS \cup PV\right). \quad (12)$$

Performing further simplification with Eq. 7, and substituting the corresponding failure probability of each component from Table 1, the system-failure probability at time t_0 = 2,160 hours would be 33.50%, which provides a reliability of 66.50%. **Fig. 7** depicts the reliability of the separator system and its components (SEP, LV1, LV2, LV3, PS, and PV) as a function of operation time. Because the failure of these components is linked to system failure by means of an OR-gate, the high failure rate of the separator (11 to 20 times more than the failure rate of valves, as presented in Table 1) has a major negative effect on the system-reliability performance. Some measures, such as condition monitoring, preventive-maintenance actions, and adding redundancy to the system, may be taken into consideration to keep the system reliability above a desired level.

Illustration of Phase II. ***Step II-1: Expert Selection***. In this study, experts are chosen on the basis of the criteria suggested by Ortiz et al. (1991). In this regard, experts collectively should represent a wide variety of backgrounds and experience. Referring to the publications of experts and their direct involvement in or consulting and managing of research in the related areas could also be a help-

Component	Component Identification	Failure Rate λ (failure/hr)	MTTF (hours)	$F(t_0)$ (%)	$R(t_0)$ (%)
Three-phase separator	SEP	1.38×10^{-4}	7.2275×10^{3}	25.83	74.17
Level-control valve—oil dump	LV1	1.22×10^{-5}	8.1833×10^{4}	2.61	97.39
Level-control valve—water dump	LV2	1.22×10^{-5}	8.1833×10^{4}	2.61	97.39
Level-control valve—inlet mixture	LV3	1.22×10^{-5}	8.1833×10^{4}	2.61	97.39
Pressure-safety (relief) valve	PS	6.93×10^{-6}	1.4430×10^{5}	1.49	98.51
Pressure-control valve—gas outlet	PV	6.93×10^{-6}	1.4430×10^{5}	1.49	98.51

Table 1—List of components, their identification, MTTF, and reliability at t0=2,160 hours.

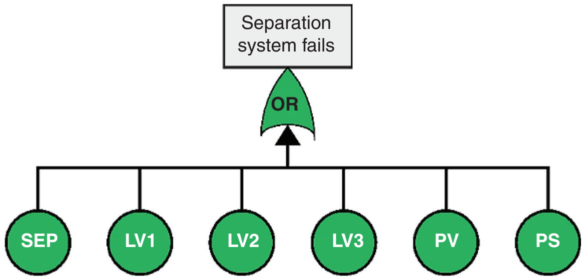

Fig. 6—Fault-tree diagram of the separator and its valves.

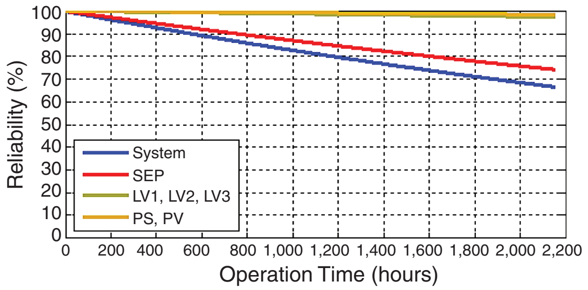

Fig. 7—Reliability of the separator system and its components.

ful basis for expert selection. In this study, the key point in expert selection is that they must have adequate understanding of Arctic operating conditions and their potential effects on the performance of mechanical systems. Considering these criteria, six experts are chosen from Norwegian academic and industrial sectors, each with appropriate knowledge of the operating conditions in the North Sea (i.e., the base area) and the Barents Sea (i.e., the target area). Selected experts have expertise in maintenance and reliability engineering, process engineering, mechanical engineering, and cold-climate engineering, with an experience ranging from 7 to 40 years.

The authors selected these experts on the basis of the available resources at the time of the study. Additionally, because the primary goal of this study is to propose the methodology and highlight the necessity of including Arctic operating conditions in system-reliability assessments, the authors decided to perform the analyses on the basis of the opinions of these selected experts, and to not exclude the experts with the working experience of 7 and 9 years. However, those two experts will receive a lower weight compared with those having more working experience. The results of this study may not be considered universal because they depend on the expert-selection criteria, number of experts, and varying operating conditions. Therefore, other studies may develop their own expert-selection schemes and achieve different results. A detailed discussion on selection and motivating experts is presented by Meyer and Booker (1991).

Step II-2: Expert-Opinion Elicitation. At the next step, a questionnaire is prepared during which experts are informed about the operating environment in the base and target areas, as well as the MTTF of valves and three-phase horizontal separator. Experts are then asked to provide their subjective opinions on the degree of decrease in such MTTFs, considering that the described equipment is planned to operate in the target area. The Johan Castberg field, which is located 230 km north of the Norwegian coast in the Barents Sea, is selected as the target area. The operating conditions in this region are much more severe compared with the southern regions of the Norwegian continental shelf, such as the North Sea. The questionnaire used in this study for the expert-elicitation step is presented in Appendix B. **Table 2** presents the elicited expert opinions, their working experience in years, and both the nonnormalized and normalized experience-based weights for the experts that will be further used for combining expert opinions. Nonnormalized weights are determined by dividing the working experience of each expert (in years) by the maximum working experience, which is 40 years. Equal weighting (w_i = 0.1667) is another approach used in this study, which is one divided by the number of experts.

Step II-3: Fuzzification of Expert Opinions. The next step is to fuzzify elicited expert opinions, through which the elicited degree of decrease in MTTF [$\delta_{i,L}$, $\delta_{i,M}$, $\delta_{i,R}$] is converted to a Gaussian fuzzy number $\tilde{\Delta}_i$. The method to determine the corresponding membership function for each expert's data is described in Appendix A. The fuzzified expert opinions on parameter δ for both the separator and valves are plotted in **Figs. 8a and 8b,** respectively. As can be seen, according to the 5, 50, and 95% quantiles of the data given by Expert 3, the MTTF of the separator in the target area may decrease by (50, 62.5, 75%). However, on the basis of the membership function of the corresponding fuzzy number, the reduction in MTTF may be as high as 81%, or as low as 43.9%, but with a membership degree of 0.05.

Step II-4: Aggregation of Fuzzified Expert Opinions. The fuzzified expert opinions are then combined by use of Eqs. 9 and 10 according to the weights assigned for each expert. For this purpose, four approaches are used in this study, as presented in **Table 3. Fig. 9** shows the combined fuzzy expert opinions by use of these four approaches. For instance, if one chooses arithmetic averaging and determines each expert's weight on the basis of their working experience (i.e., Approach II), the MTTF of the separator in the target area will be 35.2% smaller than in the base area. However, this degree of decrease is obtained if one chooses the membership grade of unity. The reduction in MTTF of the separator would be as large as 51%, but with a membership grade of 0.05.

Step II-5: Developing Fuzzy Failure-Probability Function for the Components/Subsystems. Having the changes in the MTTF of the separator and valves estimated, one can predict the reliability or failure probability of those components as a function of time. For example, **Fig. 10a** illustrates the separator reliability at t_0 = 2,160 hours in both the base and target areas. As shown in this figure, after 2,160 hours, separator reliability reduces to 74.17% in the base area. This reduction is considerably larger in the target area because of the adverse effects of Arctic operating conditions on separator performance. Additionally, as can be seen, four different approaches estimate different reductions in separator reliability, of which the greatest and the least reductions are estimated by Approaches II and IV, respectively. By use of the same procedure, the

Expert Number	Δ_i=[$\delta_{i,L}$, $\delta_{i,M}$, $\delta_{i,R}$] (fraction)		Experience (years)	Experience-Based Weights	
	Separator	Valve (general)		Nonnormalized	Normalized
1	[0, 0.125, 0.25]	[0.25, 0.375, 0.5]	9	0.225	0.0633
2	[0.5, 0.625, 0.75]	[0.5, 0.625, 0.75]	40	1.000	0.2817
3	[0.5, 0.625, 0.75]	[0.25, 0.375, 0.5]	30	0.750	0.2113
4	[0, 0.125, 0.25]	[0, 0.125, 0.25]	26	0.650	0.1831
5	[0, 0.05, 0.1]	[0, 0.05, 0.1]	7	0.175	0.0493
6	[0, 0.05, 0.1]	[0, 0.125, 0.25]	30	0.750	0.2113

Table 2—Elicited expert opinions and experts' experience-based weights.

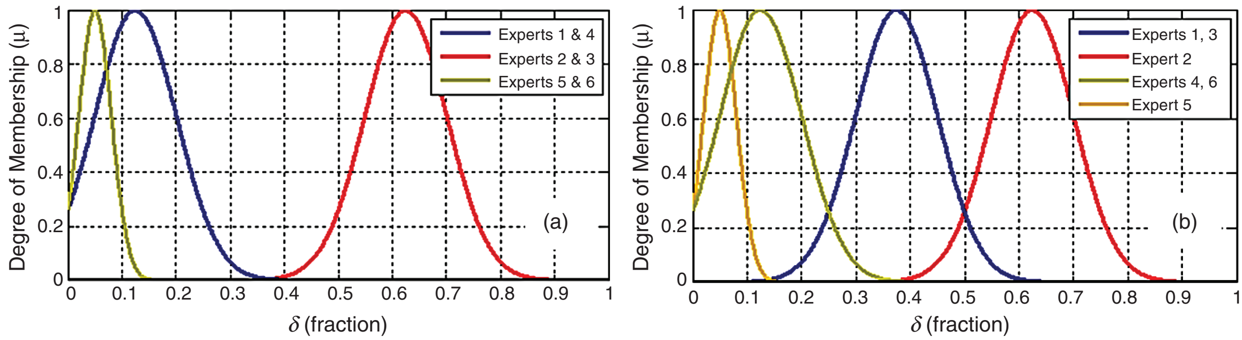

Fig. 8—Fuzzified expert data on the amount of decrease in MTTF of (a) the separator and (b) the valve, in general.

Approach	Description
I	Arithmetic averaging with equal weights
II	Arithmetic averaging with experience-based weights
III	Geometric averaging with equal weights
IV	Geometric averaging with experience-based weights

Table 3—Approaches used to combine fuzzified expert opinions.

reliability of pressure- and level-control valves can be described in fuzzy form, as well.

Step II-6: Performing Fuzzy Fault-Tree Analysis (FTA) To Estimate the System-Failure Probability. Once component reliabilities (or failure probabilities) are estimated, system performance can be analyzed by use of the fuzzified form of Eq. 12 (see Appendix A). Fig. 10b shows the fuzzy form of system reliability at time t_0 = 2,160 hours. As can be seen, the system reliability in the base area is 66.50%. Taking into account the effects of operating conditions in the target area on system performance, this reliability continues to decrease, as illustrated by Approaches I through IV. For instance, considering Approach II and membership grades of unity and 0.05, system reliability reduces to 53.58 and 43.53%, respectively. Choosing a specific membership grade and its corresponding system reliability depends on the aim of the study, requirements and regulations, and the risk perception of the decision maker. However, according to the fuzzy set theory, the higher the membership grade, the more the element belongs to the set. One may also defuzzify the results to obtain the corresponding crisp values.

In addition to the system reliability at a specified time, the proposed methodology is also applicable to developing the system reliability as a function of operation time. **Fig. 11** illustrates the system reliability in the base and target areas on the basis of four approaches and taking a membership degree of unity. Among the introduced approaches, Approaches II and III estimate the greatest and the least reduction in system reliability, respectively. However, it must be noted that these results are not universal because they may differ on the basis of various expert opinions, varying operating conditions, operation location, system production rate, type of wellstream, and available inspection and maintenance activities.

Conclusion

This study focuses on the application of expert judgement in reliability prediction of oil and gas topside facilities in Arctic regions, where adequate life data may not be available. The proposed methodology can be used in the design phase for oil and gas operations in the Arctic. The estimated reduction in reliability performance of the equipment can be used for optimizing maintenance and spare-parts provision plans. Moreover, this methodology provides a basis for deciding the measures of winterization that need to be applied if the system reliability is below the acceptable level. Additionally, as illustrated in this study, fuzzy set theory can be used to aggregate expert opinions, while including and modeling the uncertainties and their propagation in system analysis. The estimation made by the presented methodology may need to be further modified whenever new historical or laboratory life data are available.

The life data for reliability analysis in the base area are obtained from the OREDA handbook (OREDA Participants 2009), which provides only the constant failure rate of the equipment. Thus, exponential distribution is the only applicable probability-density function to estimate component reliabilities. Use of detailed life data from maintenance reports can lead to more-dependable reliability analysis.

The number of selected experts is also a determining factor in the reliability of the results. Such experts must have adequate un-

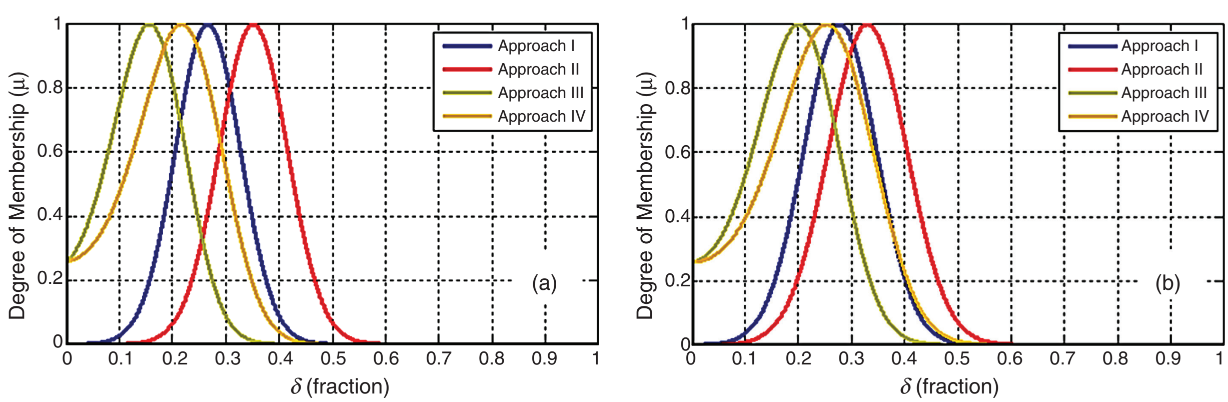

Fig. 9—Combined fuzzy expert opinions on (a) MTTF of SEP and (b) on MTTF of LV1, LV2, LV3, PS, and PV.

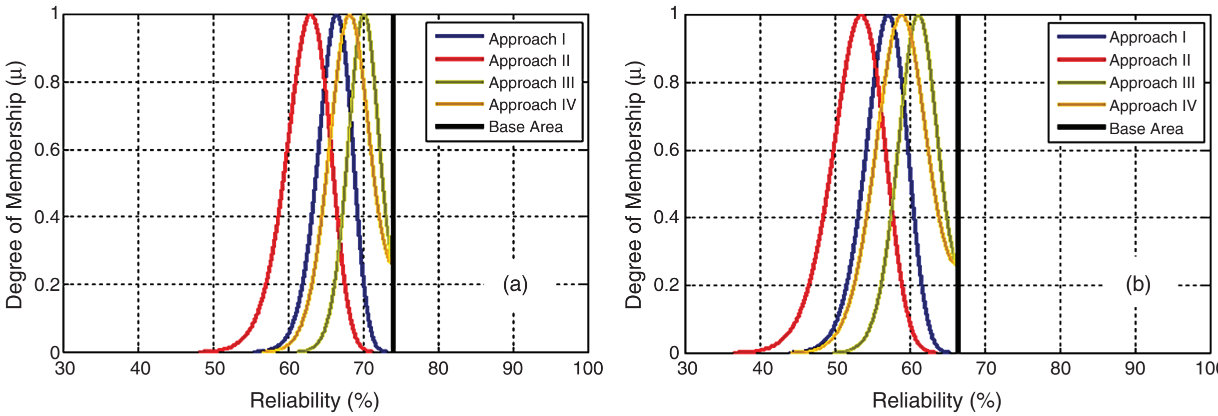

Fig. 10—Fuzzy reliability of (a) the separator and (b) the system in the base and target areas at t_0 = 2,160 hours.

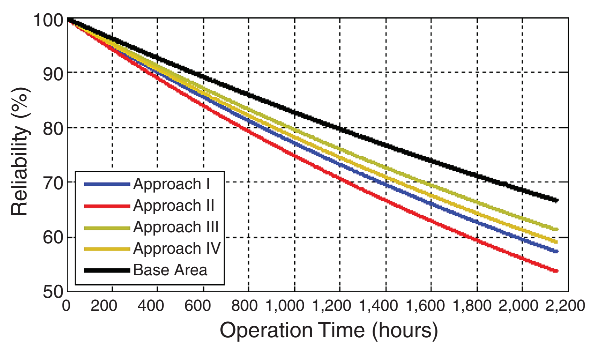

Fig. 11—System reliability in the base and target areas as a function of operation time.

derstanding of the failure mechanisms of various components and the effects of Arctic operating conditions on such mechanisms. The use of equal weights for experts may not seem realistic because one may argue various experts have different levels of expertise and thus should receive different weighting factors. In this regard, other weighting methods that can calibrate expert opinions may be more useful. Additionally, within the elicitation step, questions may be asked in different forms to ensure the consistency of expert opinions. Although one needs to establish a series of criteria for expert selection, expert selection remains a major issue in any expert-judgement process because understanding and quantifying the competence level of each expert are quite challenging tasks.

Nomenclature

$f(x)$ = probability-density function
$F(x)$ = failure probability or unreliability function
F_j = failure probability of component j
N = total number of experts
$R(x)$ = reliability function
$\bar{T}_{j,B}$ = mean time to failure (MTTF) of component j in the base area
$\bar{T}_{j,T}$ = MTTF of component j in the target area
w_i = normalized weight for Expert i
x = element of Gaussian fuzzy number $\tilde{X}$
x_M = mean value of Gaussian fuzzy number $\tilde{X}$
$\tilde{X}$ = Gaussian fuzzy number
X_α = α-cut set of Gaussian fuzzy number $\tilde{X}$
$X_{\alpha L}$ = lower bound of the α-cut set of Gaussian fuzzy number $\tilde{X}$
$X_{\alpha R}$ = upper bound of the α-cut set of Gaussian fuzzy number $\tilde{X}$
δ = combined expert opinions in crisp form; element of the Gaussian fuzzy number $\tilde{\Delta}$
δ_i = degree of decrease in MTTF of a component elicited from Expert i; element of the fuzzy number $\tilde{\Delta}_i$
$\delta_{i,L}$ = 5% quantile of the parameter δ elicited from Expert i
$\delta_{i,M}$ = 50% quantile of the parameter δ elicited from Expert i
$\delta_{i,R}$ = 95% quantile of the parameter δ elicited from Expert i
δ_{i,α_L} = lower bound of the α-cut set of Gaussian fuzzy number $\tilde{\Delta}_i$
δ_{i,α_R} = upper bound of the α-cut set of Gaussian fuzzy number $\tilde{\Delta}_i$
δ_j = combined crisp expert opinion on the reduction in MTTF of component j; element of the fuzzy number $\tilde{\Delta}_j$
δ_M = mean value of the combined fuzzified expert opinions
$\tilde{\Delta}$ = combined fuzzified expert opinions
Δ_α = α-cut set of Gaussian fuzzy number $\tilde{\Delta}$
$\tilde{\Delta}_i$ = Gaussian fuzzy number representing the quantiles given by Expert i
$\tilde{\Delta}_{i,\alpha}$ = α-cut set of Gaussian fuzzy number $\tilde{\Delta}_i$
$\tilde{\Delta}_j$ = combined fuzzified expert opinions on the reduction in MTTF of component j
$\Delta_{j,\alpha}$ = α-cut set of Gaussian fuzzy number $\tilde{\Delta}_j$
λ = failure rate
$\mu_{\tilde{X}}(x)$ = membership function of Gaussian fuzzy number $\tilde{X}$
σ_L = left-side standard deviation of a Gaussian fuzzy number
σ_R = right-side standard deviation of a Gaussian fuzzy number
$\tilde{\Psi}_j$ = fuzzified failure probability of component j
$\tilde{\Psi}_{j,\alpha}$ = α-cut set of Gaussian fuzzy number $\tilde{\Psi}_j$

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Appendix A

Gaussian Fuzzy Form of *δ*. Let the value of parameter δ be given by Expert *i* in the form of $[\delta_{i,L}, \delta_{i,M}, \delta_{i,R}]$, where $\delta_{i,L}$, $\delta_{i,M}$, and $\delta_{i,R}$ are 5, 50, and 95% quantiles, respectively. To fuzzify the parameter δ with a Gaussian fuzzy number, one needs to determine the standard deviations of the normal distributions corresponding to the elicited quantiles. This can be achieved by solving the cumulative density function of a normal distribution for $\sigma_{i,L}$ and $\sigma_{i,R}$, as given by

$$\begin{cases} \sigma_{i,L} = \left(\delta_{i,L} - \delta_{i,M}\right) / \left[\sqrt{2}\mathrm{erf}^{-1}\left(-0.9\right)\right], \text{ for } \delta < \delta_{i,M} \\ \sigma_{i,R} = \left(\delta_{i,R} - \delta_{i,M}\right) / \left[\sqrt{2}\mathrm{erf}^{-1}\left(+0.9\right)\right], \text{ for } \delta \geq \delta_{i,M} \end{cases} \quad \text{(A-1)}$$

Having $\sigma_{i,L}$ and $\sigma_{i,R}$ determined, the Gaussian membership function of the fuzzy number $\tilde{\Delta}_i = \left\{\left[\delta_i, \mu_{\tilde{\Delta}_i}\left(\delta_i\right)\right], \delta_i \in (0,1)\right\}$ can be obtained by use of Eq. 1, as

$$\mu_{\tilde{\Delta}_i}\left(\delta\right) = \begin{cases} \exp\left[-\left(\delta - \delta_{i,M}\right)^2 / \left(2\sigma_{i,L}^2\right)\right], \text{ for } \delta < \delta_{i,M} \\ \exp\left[-\left(\delta - \delta_{i,M}\right)^2 / \left(2\sigma_{i,R}^2\right)\right], \text{ for } \delta \geq \delta_{i,M} \end{cases} \quad \text{(A-2)}$$

To write the Gaussian membership function with the α-cut set concept $\Delta_{i,\alpha} = \left(\delta_{i,\alpha_L}, \delta_{i,\alpha_R}\right)$, one can equate the membership function $\mu_{\tilde{\Delta}_i}\left(\delta\right)$ with α, and solve the resulting equation for δ, as given by

$$\begin{cases} \delta_{i,\alpha_L} = \delta_{i,M} - \sqrt{-2\sigma_{i,L}^2 \ln\alpha} \\ \delta_{i,\alpha_R} = \delta_{i,M} + \sqrt{-2\sigma_{i,R}^2 \ln\alpha} \end{cases} \quad \text{(A-3)}$$

Equipment	MTTF in the base area[1]	Decrease in MTTF under Arctic operating conditions (i.e., target area) with no winterization[2]			
The field you are working in:					
Your experience, number of years:					
		Lower Bound (%)	Mean (%)	Upper Bound (%)	Comments
Three-phase oil/gas separator	10 months				
Valve (all types)	2.5 years				

[1] You may consider the North Sea as a base area.

[2] Assume an offshore platform used for separating oil and gas from Johan Castberg field in the Norwegian Barents Sea, 230 km north of the Norwegian coast. Consider the following key operating environment: Minimum temperature = –30°C; polar nights: 14 November–28 January (76 days); minimum sea-surface temperature: –2 to 0°C; maximum wave height: 15 m; maximum atmospheric icing because of precipitation (e.g., rain): 10 to 15 cm, with the possibility of having polar low pressure, rain, and a snowstorm.

Table B-1—The questionnaire used for eliciting expert opinions.

To combine the fuzzified expert data by use of the arithmetic and geometric averaging rules, the following equations can be used, respectively:

$$\tilde{\Delta} = \sum_{i=1}^{N} w_i \tilde{\Delta}_i \quad \text{(A-4)}$$

and

$$\tilde{\Delta} = \prod_{i=1}^{N} \tilde{\Delta}_i^{w_i}, \quad \text{(A-5)}$$

where $\tilde{\Delta}_i$ is fuzzified opinion of Expert i, $\tilde{\Delta}$ is the combined expert opinions in fuzzy form, N is the total number of experts, and w_i is the normalized nonnegative weight for Expert i. The membership function of the combined fuzzified expert data $\tilde{\Delta} = \left\{\left[\delta, \mu_{\tilde{\Delta}}(\delta)\right], \delta \in (0,1)\right\}$ is then obtained with the extension principle and α-cut set concept, given by

$$\Delta_\alpha = \sum_{i=1}^{N} w_i \Delta_{i,\alpha} = \left[\delta \in (0,1) \middle| \delta = \sum_{i=1}^{N} w_i \delta_i, \delta_i \in \Delta_{i,\alpha}\right] \quad \text{(A-6)}$$

and

$$\Delta_\alpha = \prod_{i=1}^{N} \Delta_{i,\alpha}^{w_i} = \left[\delta \in (0,1) \middle| \delta = \prod_{i=1}^{N} \delta_i^{w_i}, \delta_i \in \Delta_{i,\alpha}\right], \quad \text{(A-7)}$$

where $\Delta_{i,\alpha}$ is the α-cut set of the fuzzy number $\tilde{\Delta}_i$.

Fuzzified Failure Probability and Fuzzy FTA. Once the fuzzified expert opinions on the amount of decrease in the mean time to failure of component j are combined, the failure-probability function of that component can be expressed in fuzzy form by

$$\tilde{\Psi}_j = 1 - \exp\left[-\frac{t}{\left(1-\tilde{\Delta}_j\right)\bar{T}_{j,B}}\right], \quad \text{(A-8)}$$

where its membership function is obtained by use of the extension principle and the α-cut set concept as

$$\Psi_{j,\alpha} = 1 - \exp\left[-\frac{t}{\left(1-\Delta_{j,\alpha}\right)\bar{T}_{j,B}}\right]$$

$$= \left\{F_j \in (0,1) \middle| F_j = 1 - \exp\left[-\frac{t}{\left(1-\delta_j\right)\bar{T}_{j,B}}\right], \delta_j \in \Delta_{j,\alpha}\right\}. \quad \text{(A-9)}$$

To perform the fault-tree analysis on the basis of the fuzzy set theory, the failure probability of each component must be fuzzified with Eqs. A-8 and A-9. The top-event probability of the fault tree will be further estimated by fuzzifying the AND-gate and OR-gate probabilities, as given by

$$\tilde{\Psi}_{\text{AND-Gate}} = \tilde{\Psi}_1 \tilde{\Psi}_2 \quad \text{(A-10)}$$

and

$$\tilde{\Psi}_{\text{OR-Gate}} = \tilde{\Psi}_1 + \tilde{\Psi}_2 - \tilde{\Psi}_1 \tilde{\Psi}_2, \quad \text{(A-11)}$$

where $\tilde{\Psi}_j$, $j = 1,2$ is the fuzzy failure probability of component j. The membership functions of $\tilde{\Psi}_{\text{AND-Gate}}$ and $\tilde{\Psi}_{\text{OR-Gate}}$ can be obtained by use of the extension principle and the α-cut set concept:

$$\Psi_{\text{AND-Gate},\alpha} = \Psi_{1,\alpha} \Psi_{2,\alpha}$$
$$= \left[F \in (0,1) \middle| F = F_1 F_2, F_1 \in \Psi_{1,\alpha}, F_2 \in \Psi_{2,\alpha}\right] \quad \text{(A-12)}$$

$$\Psi_{\text{OR-Gate},\alpha} = \Psi_{1,\alpha} + \Psi_{2,\alpha} - \Psi_{1,\alpha}\Psi_{2,\alpha}$$
$$= \left[F \in (0,1) \middle| F = F_1 + F_2 - F_1 F_2, F_1 \in \Psi_{1,\alpha}, F_2 \in \Psi_{2,\alpha}\right] \quad \text{(A-13)}$$

Appendix B

See **Table B-1** for the questionnaire used to elicit expert opinions.

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